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# USING OF HUNGARIAN METHOD TO SOLVE THE TRANSPORTATION PROBLEMS

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## Abstract:-

*Transportation problems and Assignment Problems are considered as one of the most important applications of linear programming and are used to solve many economic and administrative problems. Transport issues are those matters that concern the transfer of certain products from the places of production or manufactured to the places of consumption or storage, through a special matrix containing figures for transport costs, in which the main objective is to make the cost of transport at a minimum value taking into account supply and demand constraints. There are several methods to solve these types of problems, where the best methods are Vogel's Approximation Method and its modifications that their algorithm is based on finding the lowest possible cost of transport. As for, the allocation issues are meant to be those issues that discuss the optimal allocation of various economic resources on the various works to be achieved, so that we achieve either the lowest possible cost to accomplish these works or the greatest possible return through the completion of these works. This type of problem can be solved by using the Hungarian Method and the algorithm of this method is based on finding the lowest cost in the case of minimization models and the greatest possible return in the case of maximization issues. This paper presents an attempt to implement the Hungarian method algorithm in case of minimization of transport issues, and ensure that it will give an optimal solution comparing to other methods that are assigned to solve these types of transport problems. Where, the results show that the suggested method gives the same or better solution than the other methods.*

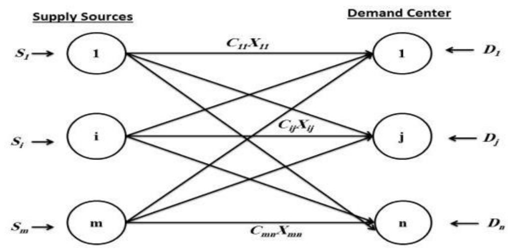
**Keywords: -** *Transportation Problems, Vogel's Approximation Method and Hungarian Method.*

## INTRODUCTION.

This paper presents a new approach to solve balanced and unbalanced transportation issues, where It is well known that the methods to solve transportation problems are the Vogel (VAM) method and its modifications. This approach is to use the Hungarian method in solving transportation models and try to ensure their effectiveness and efficiency to solve this type of issue. It will be shown that applying the Hungarian method to different examples, will gives a solution equal to the VAM's solution and sometimes may give a better one. The transportation issues model allows formulating a certain procedure for the solution called the transportation technique, which is more easily and efficiently computational, comparing to the use of the ordinary Simplex method in this regard.

The transportation model aims to achieve an optimum transportation plan for the number of units of a particular commodity from the supply sources to a number of demand centers. The model data includes the size of the available quantity in each supply source and the required quantity in each demand center, In addition to the cost of transferring one unit of the commodity from each source to the demand centers that are limited and known. The aim of using the transportation model is to determine the optimum quantities, which is needed to be transferred from each particular supply source to a particular demand center with the lowest possible cost of transportation. The model assumes that the cost of transferring on a certain path is related to the number of transferred units on this path.

The following diagram shows the transportation model as a network consisting of m supply sources and n demand centers.




Where,

- $X_{ij}$ is transferred quantity from $i^{th}$ supply source to $j^{th}$ demand centers.
- $C_{ij}$ is the cost of transferring one unit from $i^{th}$ supply source to $j^{th}$ demand centers.

Then, the general goal of this model is to minimize the target function Min Z =

$$\sum_{i=1}^{m} \sum_{nj=1} CijXij$$

Unbalanced Transportation Problems are issues where the supply's size ($\sum_{i=1}^{m} S_i$) does not match with the demand's size ($\sum_{j=1}^{n} D_j$), and this type of problems is more common in practical application than the balance issues, where, the size of the supply may exceed the size of the demand or vice versa. Before dealing with the unbalanced issues, this problem must be solved by adding a dummy supply source or dummy demand center, where the available quantity is equal to the difference between the supply total and the demand total. The transportation's cost per unit will equal zero for all cells of the added dummy row or the added dummy column, because there is no real transfer. Vogel's Approximation Method (VAM) method is considered as the most important method in solving transportation problems, which often provides an initial optimum solution or near optimum solution. The advantage of VAM's method in handling the transportation issues is to trace all minimum costs in the transportation table.

The transportation table has to be balanced, but if it is not the case a dummy row or a dummy column is needed to be added with zero cost in order to balance the problem before it can be solved. The steps of this method are:

**1.**Evaluate the penalty for each row or column by subtracting the smallest cost element in a row or a column from the smallest cost followed in the same row or column,

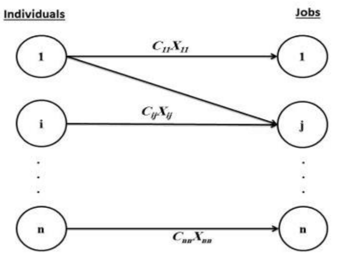
**2.**Allocate the largest possible amount to the cell that has the smallest cost in that row or column chosen based on the largest obtained penalty.

**3.**Adjust the supply and demand, and exclude the row that the demand is satisfied or the column that the supply is depleted.

**4.**Then repeat these steps again until the supply and the demand are met for all sources and agencies.

The model of allocation issues allows us to establish the certain procedures for the solution that are called the allocations technique, which is also considered as more easily and efficiently from computational point of view comparing to the application of the usual Simplex method in this regard. The allocation model aims to find out an optimum allocation plan for various economic resources on the different jobs to be accomplished. The model's data include the number of employees and the number of different jobs, whereas, each individual is assigned to achieve one job only and each job is performed by one person only, as well as, the cost of people's achievement for different works.

The following diagram shows the allocation model as a network consisting of n individuals and n jobs.




Where,

- $X_{ij}$ indicates that the $i^{th}$ person is allocated for the $j^{th}$ job. And $X_{ij}$ takes 1 if the $i^{th}$ person is allocated for the $j^{th}$ job and $X_{ij}$ takes 0 otherwise.
- $C_{ij}$ indicates the cost of achieving the $j^{th}$ job by the $i^{th}$ person.

Then, the general goal of this model is to minimize the target function Min Z =

$$\sum_{i=1}^{n} \sum nj{=}1\ CijXij$$

The Hungarian method is considered as the best method to solve the allocation issues, where its solution is based on subtracting the smallest element in each row of all the elements of that row, then subtracting the smallest element in each column of that column. Then we get the cost matrix of total opportunity matrix, which produces a zero element in each row and in each column.

The following steps should be done in order to test the optimization of the solution:

**1.** Draw the minimum number of straight lines passing through all the zero elements in the previous matrix. If the number of these straight lines zero is less than n, meaning that the solution is not optimal and needs to be developed.

**2.** Choosing the smallest element that was not passed by any line of straight lines and adding it to every element intersects the two lines and subtracting it on all the elements that were not passed by any line.

**3.** The rest of the elements that are located on the straight lines stay on their situation without any change.

**4.** Repeat the previous steps until the optimum allocation are reached in order to achieve the required number of straight lines, which must be equal to the number of tasks or number of individuals.

**Research Objective:**

This research aims to implement the Hungarian method to solve balanced and unbalanced conductor problems and to ensure whether or not it is suitable use the Hungarian method as an alternative to the VAM method in solving the transportation problems. In order to ensure the suitability of this suggestion, several examples will be solved using the VAM and the Hungarian methods and compare their results.

**Example 1: (balanced problem) suppose there is the following transportation matrix.**

| to / from | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| A | 5 | 15 | 16 | 40 |
| B | 35 | 10 | 25 | 18 |
| C | 20 | 30 | 6 | 45 |
| D | 40 | 20 | 46 | 7 |

**First: the solution using the VAM method.**

| to / from | 1 | 2 | 3 | 4 | Supply |
|---|---|---|---|---|---|
| A | 5 (10) | 15 (4) | 16 (1) | 40 | 15 |
| B | 35 | 10 (6) | 25 | 18 | 6 |
| C | 20 | 30 | 6 (14) | 45 | 14 |
| D | 40 | 20 (2) | 46 | 7 (9) | 11 |
| Demand | 10 | 12 | 15 | 9 | 49 |

Total cost of transportation = 373

**Second: the solution using the Hungarian method.**
The total opportunity matrix can be determined as following:

| from \ to | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| A | 0 | 10 | 11 | 35 |
| B | 25 | 0 | 15 | 8 |
| C | 14 | 24 | 0 | 39 |
| D | 33 | 13 | 39 | 0 |

By testing the solution, it was found to satisfy the optimum condition, where less number can be drawn from lines is n. the optimum solution gives the following allocation: A→1, B→2, C→3, D→4

| from \ to | 1 | 2 | 3 | 4 | Supply |
|---|---|---|---|---|---|
| A | 5(10) | 15(4) | 16(1) | 40 | 15 |
| B | 35 | 10 (6) | 25 | 18 | 6 |
| C | 20 | 30 | 6 (14) | 45 | 14 |
| D | 40 | 20 (2) | 46 | 7 (9) | 11 |
| Demand | 10 | 12 | 15 | 9 | 49 |

By completing the distribution of supply on demand, the transportation cost is calculated to be Z=373, which is the same cost given by VAM method.

**Example 2: (balanced problem) suppose there is the following transportation matrix.**

| from \ to | 1 | 2 | 3 | 4 | Supply |
|---|---|---|---|---|---|
| A | 18 | 13 | 14 | 10 | 48 |
| B | 16 | 17 | 8 | 15 | 80 |
| C | 12 | 15 | 18 | 13 | 72 |
| Demand | 30 | 60 | 70 | 40 | 200 |

**First: the solution using the VAM method.**

| from \ to | 1 | 2 | 3 | 4 | Supply |
|---|---|---|---|---|---|
| A | 18 | 13 | 14 | 10 | 48 |
| B | 16 | 17 | 8 | 15 | 80 |
| C | 12 | 15 | 18 | 13 | 72 |
| Demand | 30 | 60 | 70 | 40 | 200 |

**Second: the solution using the Hungarian method.**
First, a dummy row is added to balance the matrix as an allocation problem, while it is a balance transportation problem.

| from \ to | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| A | 18 | 13 | 14 | 10 |
| B | 16 | 17 | 8 | 15 |
| C | 12 | 15 | 18 | 13 |
| Dummy | 0 | 0 | 0 | 0 |

The total cost matrix becomes as follows:

| from \ to | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| A | 8 | 2 | 4 | 0 |
| B | 8 | 9 | 0 | 7 |
| C | 0 | 3 | 6 | 4 |
| Dummy | 0 | 0 | 0 | 0 |

By testing the solution, the optimum allocation becomes as follows: A→4, B→3, C→1, and Dummy→2

| to / from | 1 | 2 | 3 | 4 | Supply |
|---|---|---|---|---|---|
| A | 18 | 13 (8) | 14 | 10 (40) | 48 |
| B | 16 | 17 (10) | 8 (70) | 15 | 80 |
| C | 12 (30) | 15 (42) | 18 | 13 | 72 |
| Demand | 30 | 60 | 70 | 40 | 200 |

The total cost of transportation is 2224, which is the same result given by usual VAM method.

**Example 3: (unbalanced problem) suppose there is the following transportation matrix.**

| to / from | 1 | 2 | 3 | 4 | 5 | Supply |
|---|---|---|---|---|---|---|
| A | 25 | 23 | 22 | 28 | 26 | 160 |
| B | 15 | 25 | 18 | 23 | 24 | 240 |
| C | 22 | 18 | 20 | 25 | 22 | 320 |
| D | 24 | 22 | 21 | 20 | 23 | 280 |
| Demand | 200 | 260 | 280 | 220 | 140 | 1000 / 1100 |

It can be noticed that the supply amount is less than the amount of demand, so it is needed to be balance by adding a dummy row contains the amount equal 100, and then use the VAM method as usual.

**First: the solution using the VAM method.**

| to / from | 1 | 2 | 3 | 4 | 5 | Supply |
|---|---|---|---|---|---|---|
| A | 25 | 23 | 22 (160) | 28 | 26 | 160 |
| B | 15 (200) | 25 | 18 (40) | 28 | 24 | 240 |
| C | 22 | 18 (260) | 20 (60) | 25 | 22 | 320 |
| D | 24 | 22 | 21 (20) | 20 (220) | 23 (40) | 280 |
| Dummy | 0 | 0 | 0 | 0 | 0 (100) | 100 |
| Demand | 200 | 260 | 280 | 220 | 140 | 110 |

The total cost of transportation is equal to 18860

**Second: the solution using the Hungarian method.**

The total opportunity matrix can be determined as following:

| to / from | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| A | 3 | 1 | 0 | 6 | 4 |
| B | 0 | 10 | 3 | 8 | 9 |
| C | 4 | 0 | 2 | 7 | 4 |
| D | 4 | 2 | 1 | 0 | 3 |
| Dummy | 0 | 0 | 0 | 0 | 0 |

By using the optimum allocation in this matrix, the following solution is gotten: A→3, B→1, C→2, D→4, Dummy→5 and the optimum distribution that was found from using the Hungarian method, is given as follows:

| to / from | 1 | 2 | 3 | 4 | 5 | Supply |
|---|---|---|---|---|---|---|
| A | 25 | 23 | 22 (160) | 26 | 26 | 160 |
| B | 15 (200) | 25 | 18 (40) | 28 | 24 | 240 |
| C | 22 | 18 (260) | 20 (60) | 25 | 22 | 320 |
| D | 24 | 22 | 21 (20) | 20 (220) | 23 (40) | 280 |
| Dummy | 0 | 0 | 0 | 0 | 0 (100) | 100 |
| Demand | 200 | 260 | 280 | 220 | 140 | 110 |

The total cost of transportation is equal to 18860, which is the same result given by usual VAM method.

**Example 4: (unbalanced problem)** let consider the following transportation matrix.

| to | 1 | 2 | 3 | 4 | Supply |
|---|---|---|---|---|---|
| | 150 | 80 | 100 | 180 | 250 |
| B | 130 | 170 | 180 | 150 | 150 |
| C | 250 | 225 | 150 | 110 | 100 |
| Demand | 120 | 150 | 180 | 110 | 550 |

It can be noticed that the supply amount is less than the amount of demand, so it is needed to be balance by adding a dummy row contains the amount equal 50, and then use the VAM method as usual.

**First: the solution using the VAM method.**

| to / from | 1 | 2 | 3 | 4 | Supply |
|---|---|---|---|---|---|
| A | 150 | 80 (150) | 100(100) | 180 | 250 |
| B | 130 (70) | 170 | 180 (80) | 150 | 150 |
| C | 250 | 225 | 150 | 110(100) | 100 |
| Dummy | 0(50) | 0 | 0 | 0 | 50 |
| Demand | 120 | 150 | 180 | 110 | 550 |

The total cost of transportation is equal to 65500.

**Second: the solution using the Hungarian method.**

Now, the Hungarian method is applied to solve this problem and find the total cost matrix after adding dummy row in order to balance this problem.

| to / from | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| A | 70 | 0 | 20 | 100 |
| B | 0 | 40 | 50 | 20 |
| C | 140 | 115 | 40 | 0 |
| Dummy | 0 | 0 | 0 | 0 |

**By testing the solution, the optimum allocation becomes as follows: A→2, B→1, C→4, and Dummy→4**

| to / from | 1 | 2 | 3 | 4 | Supply |
|---|---|---|---|---|---|
| A | 150 | 80 (150) | 100 (100) | 180 | 250 |
| B | 130 (120) | 170 | 180 (30) | 150 | 150 |
| C | 250 | 225 | 150 | 110 (100) | 100 |
| Dummy | 0 | 0 | 0 (50) | 0 | 50 |
| Demand | 120 | 150 | 180 | 110 | 550 |

The total cost of transportation is equal to 64500.

It can be seen that using the Hungarian method gives solution is better than the solution of the usual VAM method.

**Conclusion:**

From the introduction and the previous examples, it can clearly be seen that the Hungarian method can be used to find an initial solution to balanced and unbalanced transportation issues instead of the usual VAM method. Where, the Hungarian method, when were used to solve transportation problems, provides an initial solution equal to the usual VAM method and sometimes,may give a better solution than the VAM method as it has be seen in Example 4.

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