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ANALYSIS OF A PREY – PREDATOR MODEL WITH GENERAL INCIDENCE

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Abstract:-
In this paper we have analyzed prey-predator systems where the prey population is divided into two groups, infected. Also we have considered the effect of intraspecific competition between infected preys as well as on predator where the species observed in nature species does not exist alone.
In this paper we discuss different systems of prey – predator model of the general 2dimensional with general incidence H(S, I). Also, study discuss different systems of a prey – predator model with general incidence $H(S,I)$. The aim of this paper is to study the dynamical behavior of a prey – predator model by different techniques with generalized incidence term.

Keywords:- S*tability-Lyapunove function – prey- predator – dynamic-equilibria.*

INTRODUCTION

The Lotka (1925) – Voltera (1931) model is one of the earliest prey – predator models

$$\bar{X}=X(\beta-\alpha Y)$$
$$\bar{Y}=-Y(\gamma X-\delta)$$

For these models several prey – predator models were discussed by many authors [,1 2, 3, 4, 6 and 7].
In this paper we have analyzed this model for general incidence $H(S,I)$
Data also studied nature of equilibria and stability properties and we present the following model

$$\left.\begin{aligned} \frac{ds}{dt} &= S\left\{r\left(1-\frac{S+I}{K}\right)\right\} - IH(S,I) \\ \frac{dI}{dt} &= IH(S,I) - I(C+Py+aI) \\ \frac{dy}{dt} &= y\{-d+qpI-by\} \end{aligned}\right\} \quad (1.1)$$

Where:

$H(S,I)$: general incidence rate
K: the carrying capacity of the environment
r: the intrinsic birth rate
c, d: the death rate of infected prey and predator respectively
a, b: the intraspecific competition coefficient the infected prey and predator respectively
q: the coefficient in conversing prey into predator
P: the predation coefficient
$S(t)$,$I(t)$,$y(t)$: are the population density of the susceptible prey, infected prey and predator respectively at a given time t

EXISTENCE BOUNDEDNESS

In this section, we first show that solution of system (1.1) is bounded.
Theorem 1: system (1.1) is dissipative.
Proof: let $(S(t), I(t), y(t))$ be any solution with positive initial conditions (S_0, I_0, y_0).

Since, $\frac{ds}{dt} \le sr(1-\frac{s}{k})$

We have lim sup $S(t) \le M$ (see [5]) where $M_{t\to\infty} = max\{S(0), K\}$ consider the function.

$$W = S + I + Y$$

The time derivative a long a solution of (1) is:

$$\frac{dw}{dt} = \frac{ds}{dt} + \frac{dI}{dt} + \frac{dy}{dt} \quad (2.1)$$

Therefore,

$$\frac{dw}{dt} = S\left\{r\left(1-\frac{S+I}{K}\right)\right\} - I(C+Py+\propto I) + y(-d+qpI-by) \le S(r+1) - S - cI - dy \le M(r+1) - mw \text{ (see [14])}$$

Where $m = min\{1, c, d\}$

Thus $\frac{dw}{dt} + mw \le M(r+1)$

$$0 \le w(S,I,y) \le \frac{M(r+1)}{m} + \frac{W(S(0),I(0)y(0)}{e^{mt}} \quad (2.2)$$

Therefore, all solutions of system (1.1) interinto the region

$$B = \left\{(S,I,y) \in R_+^3 : \le \frac{M(r+1)}{m} + \varepsilon forany\ \varepsilon > 0\right\} \quad (2.3) \text{ (see [4,5]).}$$

The model
Equation (1) has the following non-negative equilibria namely.

$$E_0 = (0,0,0), E_1(K,0,0), E_{1,2} = (S^*, I^*, 0)$$

The interior equilibrium point $E^*(S^*,I^*,y^*)$where

$$S^* = \frac{I(aI+C)}{r}$$

$$I^* = \frac{I(aI+C)-c}{a + r\frac{(aI+C)}{k} + \frac{(aI+C)}{rk}}$$

$$y^* = \frac{-d+qpI}{b}$$

THE NATURE OF EQUILIBRIA

In this section we study and discuss stability properties of the equilibria
The Jacobian matrix of (1.1) around $E^*$is

$$\begin{pmatrix} \frac{-2rs^*}{K} & -\frac{r+K(H_s+H_{s^*})}{K} & 0 \\ 0 & I^*H_I(S,I) - I^*H_{I^*}(S,I) - 2aI^* & -pI^* \\ 0 & qpy^* & -2by^* \end{pmatrix} \quad (3.1)$$

Since $E0$= (0,0,0) is unstable, $E1$ is unstable if $H(S,I)>c$. $E1,2$ is globally asymptotically, thus it is feasible.in the $S-I$plane. $E*$is locally asymptotically stable. We now show that $E*$is globally asymptotically stable whenever it exists.

Theorem 2. If $E*$is feasible then it is globally asymptotically stable.

Proof. Define a Lyapunov function $V(S,I,Y)$ such that $V(S,I,Y)=c1(S-S*-S*\ln SS*/) + C2(I-I*\ln II*/)+C3(y-y*-y* \ln yy*/$ Where $Ci,i$are positive constants? Evidently $V$is a positive definite function in the region B except at $E*$where it is zero. Calculating the rate of change of Valong the solutions of system (1), we get

$$\frac{dV}{dt} = C_1(S-S^*)\frac{S}{S} + C_2(I-I^*)\frac{I}{I} + C_3(y-y^*)\frac{y}{y}$$

$$= C_1(S-S^*)\left\{r\left(1-\frac{S+I}{K}\right) - \frac{IH(S,I)}{S}\right\}$$

$$+C_2(I-I^*)\{H(S,I) - C - py - aI\}$$

$$+C_3(y-y^*)\{-d + qpI + by\}$$

$$= C_1(S-S^*)\left\{\frac{-r}{K}(S-S^*) - (\frac{r}{K} + \frac{H(S,I)}{S}(I-I^*)\right\}$$

$$+C_2(I-I^*)\left\{\frac{H(S,I)}{S}(S-S^*) - p(y-y^*) - a(I-I^*)\right\}$$

$$+C_3(y-y^*)\{qp(I-I^*) - b(y-y^*)\}$$

$$\text{Choosing } C_2\frac{H(S,I)}{S} - C_1\left(\frac{r}{K} + \frac{H(S,I)}{S}\right) = 0, C_3q - C_2 = 0$$

If follows that

$$\frac{dV}{dt} = -C_1(S,S^*)^2 - C_2a(I-I^*)^2 - C_3b(y-y^*)^2 \quad (3.2)$$

And hance V is negative. So largest invariant set at
Which V=0 is the equilibrium point and by Lasalle's invariance principle,$E*$is globally asymptotically stable

CONCLUSION

Our main results are concerned for discussing equilibria, stability existence and boundedness with general incidence. $H(S,I)$ *we obtained some of the important results are the model* (1.1)E_0 is that in the absence of prey. E_1 is unstable. It is observed that If $H(S,I)$>c, the boundary equilibriaum.$E_{1,2}$is feasible. More over observation about $E_{1,2}$that it moves a higher level in (S) direction and $E_{1,2}$will imply that E_*will also be globally asymptotically stable in the S-I plane. Another important. All solution coverage to the positive equilibrium.

REFERENCES

[1].Freedman H.I "A model of predator-prey dynamics as modified by the action of a parasite", Math. Biosci. , 99,p. 143-155, 1990

[2].AtulJohri,Trivedi,Sicodiya,Sing,Jain" study of a prey- predator Model with diseased prey"MathBiosci, 13, P. 10-115, 2012.

[3].Hadeler K.P., Freedman H.I. "Predator-prey populations with parasite infec-tion",J. Math. Biol., 27, p. 609-631, 1989

[4].D. Mukherjee, Persistance and Global stability of a population in a polluted environment with deley.J.Biol. Sys. 10(1) (2002), 225-232.

[5].D. Mukherjee, stability Analysis of a prey- predator predator Model with diseased prey"MathBiosci, 13, P. 10-115, 2003

[6].Xiao Y., Chen L "Modeling and analysis of a predator-prey model with disease in the prey ", Math. Biosci., 171, p. 59-82, 2003

[7].Xiao Y., Chen L "Analysis of a three species Eco-Epidemiological model",J. Math. Anal. Appl, 258, p. 733-754, 2001