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# SEMIGROUPS IN TERMS OF INTUITIONISTIC FUZZY BI-IDEALS

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## Abstract:-

*Since Zadeh introduced fuzzy sets in 1965, a lot of new theories treating imprecision and uncertainty have been introduced. Some of these theories are extensions of fuzzy set theory. The concept of 'intuitionistic fuzzy set' (IFS) was introduced by Atanassov as a generalization of the concept fuzzy set by gives both a degree of membership and the degree of non-membership. As for fuzzy sets, the degree of membership is a real number between 0 and 1. This is also the case for the degree of non-membership, and further the sum of these two degrees is not greater than 1. Since fuzzy bi-ideal play an important role in the study of smigroup structures. The purpose of this paper is to initiate and study the intuitionistic fuzzification on the concept of several ideals in a semigroups S and investigate the basic theorem of intuitionistic fuzzy bi-ideals and discuss the relationships of left (resp. right and completely regular) semigroups in terms of intuitionistic fuzzy biideals. For any homomorphisim f from a semigroup S to semigroup T if* $B = (\mu_B, \nu_B)$ *is an intuitionistic fuzzy bi-ideal of T, then the preimage* $f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\nu_B))$ *of B under f is an intuitionistic fuzzy bi-ideal of semigroup S.*

**Keywords: -** *Semigroup, intuitionistic fuzzy set, intuitionistic fuzzy left (res. right) ideal, intuitionistic fuzzy bi-ideal, regular and intra-regular semigroups.*

## INTRODUCTION

A semigroup is an algebraic structure consisting of a non-empty set $S$ together with an associative binary operation [1]. The formal study of semigroups began in the early 20th century. Semigroups are important in many areas of mathematics, for example, coding and language theory, automata theory, combinatorics and mathematical analysis. The concept of fuzzy sets was introduced by [2] in his classic paper. Azirel Rosenfeld [3] used the idea of fuzzy set to introduce the notions of fuzzy subgroups. Nobuaki Kuroki [4, 5, 6] is the pioneer of fuzzy ideal theory of semigroups. The idea of fuzzy subsemigroup was also introduced by Kuroki [4, 6]. In [5], Kuroki characterized several classes of semigroups in terms of fuzzy left, fuzzy right and fuzzy bi-ideals. Others who worked on fuzzy semigroup theory, such as X.Y. Xie [7, 8], Y.B. Jun [9, 10], are mentioned in the bibliography. The notion of intuitionistic fuzzy sets was introduced by Atanassov [11, 12] as a generalization of the notion of fuzzy sets. The concept of (1, 2) ideals in semigroups was introduced by $S$. Lajos [13]. In this paper, we consider the semigroup $S$ in terms of intuitionistic fuzzy bi-ideals, and discuss some relations between the fuzzy subsemigroups (fuzzy bi-ideals, fuzzy left (right) ideals, fuzzy ideals, fuzzy (1, 2)- ideals of S and the subsets of $S$. Among other results we obtain some characterization theorems of regular and intra-regular semigroups in terms of intuitionistic fuzzy bi-ideals. Also for any homomorphisim $f$ from a semigroup $S$ to semigroup $T$ if $B = (\mu_B, v_B)$ is an intuitionistic fuzzy bi-ideal of $T$**,** then the preimage $f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(v_B))$ of $B$ under $f$ is anintuitionistic fuzzy bi-ideal of semigroup $S$.

## PRELIMINARIES

First we give the concept of intuitionistic fuzzy set defined by Atanassov as a generalization of the concept of fuzzy set given by Zadeh.

**Definition 1** [1, 14] If $(S,*)$ is a mathematical system such that $\forall\ a, b, c \in S$, $(a * b) * c = a * (b * c)$, then $*$ is called associative and $(S,*)$ is called a semigroup.

**Definition 2** [1, 14] A semigroup $(S,*)$ is said to be commutative if for all $a, b \in S$, $a * b = b * a$.

**Definition 3** [15] A semigroup $S$ is said to be left (right) regular if, for each element a of $S$, there exists an element$x$ in $S$ such that $a = xa^2$(resp. $a = a^2x$).

**Definition 4** [15] A semigroup $S$ is called intra-regular if for each element a of $S$, there exist elements $x, y \in S$ such that $a = xa^2y$.

**Definition 5** [15] A semigroup $S$ is called regular if for each element a of $S$, there exists an element $x \in S$ such $thata = axa$.

**Definition 6** [15] A semigroup $S$ is said to be (2, 2)-regular if$x \in x^2Sx^2$ for any $x \in S$ 909

**Definition 7** [14] A subsemigroup of a semigroup $S$ is a non-empty subset $A$ of $S$ such that $A^2 \subseteq A$

**Definition 8** [14] A left (right) ideal of a semigroup $S$ is a non-empty subset $A$ of $S$ such that $SA \subseteq A$ ($AS \subseteq A$). If $A$ is both a left and a right ideal of a semigroup $S$, then we say that $A$ is an ideal of $S$.

**Definition 9** [14] A subsemigroup $A$ of a semigroup $S$ is called a bi-ideal of $S$ if $ASA \subseteq A$.

**Definition 10** [14] A subsemigroup $A$ of $S$ is called a (1, 2)-ideal of $S$ if $ASA^2 \subseteq A$**.**

**Definition 11** [16] A fuzzy subset of a non-empty set $X$ is a function $\mu$**:** $X \to [0, 1]$.

**Definition 12** [16] A non-empty fuzzy subset $\mu$ of a semigroup $S$ is called a fuzzy subsemigroup of $S$ if $\mu(xy) \geq min\{\mu(x), \mu(y)\}\ \forall\ x, y \in S$.

**Definition 13** [16] A fuzzy subsemigroup$\mu$ of a semigroup $S$ is called a fuzzy bi-ideal of $S$ if $\mu(xyz) \geq min\{\mu(x), \mu(z)\}\ \forall\ x, y, z \in S$.

**Definition 14** [16] A fuzzy subsemigroup $\mu$ of a semigroup $S$ is called a fuzzy (1, 2)ideal of $S$ if $\mu(xw(yz)) > min\{\mu(x), \mu(y), \mu(z)\}\ \forall\ x, w, y, z \in S$.

**Definition 15** [16] A non-empty fuzzy subset $\mu$ of a semigroup $S$ is called a fuzzy left(right) ideal of $S$ if $\mu(xy) \geq \mu(y)(resp.\ \mu(xy) \geq \mu(x))\ \forall\ x, y \in S$.

**Definition 16** [16] A non-empty fuzzy subset $\mu$ of a semigroup $S$ is called a fuzzy twosided ideal or a fuzzy ideal of $S$ if it is both a fuzzy left and a fuzzy right ideal of $S$.

**Definition 17** [11,12] The intuitionistic fuzzy sets defined on a non-empty set $X$ as objects having the form

$A = \{\langle x, \mu_A(x), v_A(x)\rangle : x \in X\}$,

where the functions $\mu$: $X \to [0,1]$ and $v$: $X \to [0,1]$ denote the degrees of membership and of non-membership of each element $x \in X$ to the set $A$, respectively, and $0 \leq \mu_A(x) + v_A(x) \leq 1$ for all $x \in X$. For the sake of simplicity, we shall use the symbol $\langle \mu_A, v_A\rangle$ for the intuitionistic fuzzy set $A = \{\langle x, \mu_A(x), v_A(x)\rangle : x \in X\}$.

**Definition 18** [11,12] Let $X$ be a nonempty set and let $A = \langle \mu_A, v_A\rangle$ and $B = \langle \mu_B, v_B\rangle$ be IFSs of $X$. Then

(1) iff $\mu_A(x) \leq \mu_B(x)$ and $v_A(x) \geq v_B(x)$, $A \subseteq B$

*(2)* if and only if $A \subseteq B$ and $B \subseteq A, A = B$

*(3)*, $A^c = \{\langle x, v_A(x), \mu_A(x)\rangle, x \in S\} = \langle v_A, \mu_A\rangle$

*(4)* $A \cap B = \{\langle x, min\{\mu_A(x), \mu_B(x)\}, max\{v_A(x), v_B(x)\} : x \in S\rangle\} = \langle \mu_A \wedge \mu_B, v_A \vee v_B\rangle$,

*(5)*, $A \cup B = \{\langle x, max\{\mu_A(x), \mu_B(x)\}, min\{v_A(x), v_B(x)\} : x \in S\rangle\} = \langle \mu_A \vee \mu_B, v_A \wedge v_B\rangle$

$A = \langle \overline{v_A}, v_A\rangle = \langle 1 - v_A, v_A\rangle$. (6) $\Box A = \langle \mu_A, \overline{\mu_A}\rangle = \langle \mu_A, 1-\mu_A\rangle$ (7) $A \circ B = \langle \mu_{A\circ B}, v_{A\circ B}\rangle$ where:

$$\mu_{A\circ B}(a) = \begin{cases} \vee_{a=yz}\ min\{\mu_A(y), \mu_B(z)\} & if\ \ a = yz \\ 0 & otherwise \end{cases}$$

$$v_{A\circ B}(a) = \begin{cases} \wedge_{a=yz}\ max\{v_A(y), v_B(z)\} & if\ \ a = yz \\ 0 & otherwise. \end{cases}$$

**Definition 19** [7] Let $\{A_i\}_{i\in J}$ be an arbitrary family of IFSs in $X$, where $A_i = (\mu_{A_i}, v_{A_i})$ for each $i \in J$. Then
(1) $\cap A_i = (\wedge \mu_{A_i}, \vee v_{A_i})$.

(2) $\cup A_i = (\vee \mu_{A_i}, \wedge v_{A_i})$.

**Definition 20** [16] Let $A$ be a non-empty subset of a semi group $S$, the intuitionistic characteristic function $X_A = \langle \mu_{X_A}, v_{X_A} \rangle$ is defined as:

$$\mu_{X_A}(x) = \begin{cases} 1 & if x \in A \\ 0 & x \notin A \end{cases} \quad \text{and} \quad v_{X_A}(x) = \begin{cases} 0 & if x \in A \\ 1 & x \notin A \end{cases}$$

**INTUITIONISTIC FUZZY BI-IDEALS**
In what follows, let $S$ denote a semigroup unless otherwise specified.

**Definition 1.[16]** Let $S$ be a semi group. An intuitionistic fuzzy set $A$=$\langle \mu_A, v_A \rangle$ of $S$is said to be an intuitionistic fuzzy subsemigroup of $S$(in short, IFSS) of $S$ if $\forall\ x, y \in S$

$$\text{(i) } \mu_A(xy) \geq inf\{\mu_A(x), \mu_A(y)\},$$

$$\text{(ii) } v_A(xy) \leq sup\{v_A(x), v_A(y)\}.$$

Example 2Consider the $S$=(ℤ,+), let $E$be the set of all even integer, and $O$be the set of all odd integer.

Define $\mu_A(x): S \to [0,1]$ by $\mu_A(x) = \begin{cases} 0.4 & if\ x \in E \\ 0.2 & if\ x \in O \end{cases}$,

and define $v_A(x): S \to [0,1]$ by $v_A(x) = \begin{cases} 0.3 & if\ x \in E \\ 0.7 & if\ x \in O \end{cases}$.

Then $A = \langle \mu_A, v_A \rangle$ is an intuitionistic fuzzy subsemigroup of $S$.

Definition 3 [17] Let$S$be a semigroup. An intuitionistic fuzzy set $A$=$\langle \mu_A, v_A \rangle$to be an intuitionistic fuzzy left (right) ideal of $S$ if $\forall\ x, y \in S$

$$\text{(i) } \mu_A(xy) \geq \mu_A(y)(\text{ resp. } \mu_A(xy) \geq \mu_A(x))$$

$$\text{(ii) } v_A(xy) \leq v_A(y)(\text{resp. } v_A(xy) \leq v_A(x))$$

Both an intuitionistic fuzzy left ideal and an intuitionistic fuzzy right is called an intuitionistic fuzzy ideal (in short, IFI). It is clear that any intuitionistic fuzzy left (right) ideal of $S$ is an intuitionistic fuzzy subsemigroup of $S$.

**Example 4** Let $S = \{a, b, c\}$ be a semigroup with the following Cayley table:

| · | [illegible] | | |
|---|---|---|---|
| | [illegible] | | |
| | [illegible] | | |
| | [illegible] | | |

Define an IFS $A = \langle \mu_A, v_A \rangle$ in $S$ by $\mu_A(a) = 0.2, \mu_A(b) = \mu_A(c) = 0.5, v_A(a) = 0.5, v_A(b) = v_A(c) = 0.3$. It is clear $A = \langle \mu_A, v_A \rangle$

Definition 5[17] Let $S$ be a semigroup. An intuitionistic fuzzy set $A$=$\langle \mu_A, v_A \rangle$to be an intuitionistic fuzzy bi-ideal of $S$ if $\forall\ x, y, z \in S$

$$\text{(i) } \mu_A(xzy) \geq inf\{\mu_A(x), \mu_A(y)\}$$

$$\text{(ii) } v_A(xzy) \leq sup\{v_A(x), v_A(y)\}.$$

Example 6Let $S = \{a,b,c,d,e\}$be a semigroup with the following Cayley table:

| $\cdot$ | | | | | |
|---|---|---|---|---|---|
| $a$ | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |

Define an IFS $A = \langle \mu_A, \nu_A \rangle$ in $S$ by $\mu_A(a) = 0.9,\ \mu_A(b) = 0.8, \mu_A(c) = 0.7, \mu_A(d) = 0.7,\ \mu_A(e) = 0.6, \nu_A(a) = \nu_A(b) = 0.6,\ \nu_A(c) = 0.7,\ \nu_A(d) = 0.8,\ \nu_A(e) = 0.9.$ By routine calculation, it is clear $A = \langle \mu A, \nu A \rangle$is an intuitionistic fuzzy bi-ideal of $S$,

**Lemma 7[16]** Let $A$ be a non-empty subset of a semigroup $S$. Then

(1) $A$ is a subsemigroup of $S$ if and only if $X_A = \langle \mu_{X_A}, \nu_{X_A} \rangle$ is an intuitionistic fuzzy subsemigroup of $S$.

(2) $A$is a bi-ideal of $S$if and only if $X_A = \langle \mu_{X_A}, \nu_{X_A} \rangle$ is an intuitionistic fuzzy bi-ideal of $S$.

**Lemma 8 [16]** Let $S$ be a semigroup and $A, B \subseteq S$ then

(i) $A \subseteq B$ if and only if $X_A \subseteq X_B$.

(ii) $X_A \circ X_B = X_{A \circ B}$.

**Theorem 3.9[15]** Every intuitionistic fuzzy left (right) ideal is an intuitionistic fuzzy bi-ideal of $S$

**Proof.** Let $A = \langle \mu_A, \nu_A \rangle$ be an intuitionistic fuzzy left ideal of $S$ and $x, y, z \in S$. Then

$$\mu_A(xzy) = \mu_A((xz)y) \geq \mu_A(y) \geq inf\{\mu_A(x), \mu_A(y)\},$$

and

$$\nu_A(xzy) = \nu_A((xz)y) \leq \nu_A(y) \leq sup\{\nu_A(x), \nu_A(y)\}.$$

Thus $A = \langle \mu_A, \nu_A \rangle$ is an intuitionistic fuzzy bi-ideal of $S$. The right case is proved in an analongous way.

**Lemma 10 [16]** A semigroup is $S$ is regular if and only if $A \circ B = A \cap B$ for eachintuitionistc fuzzy right ideal $A$ and each intuitionistc fuzzy left ideal $B$ of $S$.

**Lemma 11 [16]** A semigroup is $S$ is intra regular if and only if $A \circ B \subseteq A \cap B$ for each intuitionistc fuzzy right ideal $A$ and each intuitionistc fuzzy left ideal $B$ of $S$.

**Definition 12 [16]**An intuitionistic fuzzy bi-ideal $A = \langle \mu_A, \nu_A \rangle$ of $S$ is called idempotent if

$$A = A^2 = A \circ A, \text{ that is, } \mu_{A \circ A} = \mu_A \circ \mu_A = \mu_A\ ,\ \nu_{A \circ A} = \nu_A \circ \nu_A = \nu_A.$$

**Definition 13** [16] Let $S$ be a semigroup and $A = \langle \mu_A, \nu_A \rangle$ an intuitionistic fuzzy biideal of $S$. Then $A$ is called an intuitionistic fuzzy irreducible (resp. strongly irreducible) bi-ideal of $S$ if:
For any intuitionistic fuzzy bi-ideals $B = \langle \mu_B, \nu_B \rangle$ and $C = \langle \mu_C, \nu_C \rangle$ of $S$, $B \cap C = A$ (resp. $B \cap C \subseteq A$) implies $B = A$ or $C = A$ (resp. $B \subseteq A$ *or* $C \subseteq A$).

**Lemma 14 [16]** A bi-ideal $B = \langle \mu_B, \nu_B \rangle$ of a semigroup $S$ is an irreducible (resp. strongly irreducible) if and only if the intuitionistic characteristic function $X_B = \langle \mu_{XB}, \nu_{XB} \rangle$ of $B$ is an intuitionistic fuzzy irreducible (resp. strongly irreducible) bi-ideal of $S$.

**Theorem 15 [17]** Let $S$ be a regular semigroup. If every bi-ideal of $S$ is a right(left) ideal of $S$, then every intuitionistic fuzzy bi-ideal of $S$ is an intuitionistic fuzzy right(left) ideal of $S$.

**Proof.** Assume that every bi-ideal of $S$ is a right ideal of $S$. Let $A = \langle \mu_A, \nu_A \rangle$ be an intuitionistic fuzzy bi-ideal of $S$ and let $x, y \in S$. Then $xSx$ is a bi-ideal of $S$, and so $xSx$ is a right ideal of $S$. Since $S$ is regular, we have $xy \in (xSx)S \subseteq xSx$, which implies that $xy = xzx$ for some $z \in S$. Since $A = \langle \mu_A, \nu_A \rangle$ is an intuitionistic fuzzy bi-ideal of $S$, it follows that:

$$\mu_A(xy) = \mu_A(xzx) \geq inf\{\mu_A(x), \mu_A(x)\} = \mu_A(x),$$

and

$$\nu_A(xy) = \nu_A(xzx) \leq sup\{\nu_A(x), \nu_A(x)\} = \nu_A(x).$$

Hence $A = \langle \mu_A, \nu_A \rangle$ is an intuitionistic fuzzy right ideal of $S$.

**Theorem 16** [17] If $\{A_i\}_{i \in \Lambda}$ is a family of intuitionistic fuzzy bi-ideals of $S$, then $\cap A_i$ is an intuitionistic fuzzy bi-ideal of $S$, where $\cap A_i = \langle \wedge \mu_{Ai}, \vee \nu_{Ai} \rangle$ and

$$\wedge \mu_{Ai}(x) = \inf\{\mu_{Ai}(x) |\ i \in \Lambda,\ x \in S\},$$

$$\vee\, \nu_{Ai}(x) = \sup\{\nu_{Ai}(x) \mid i \in \Lambda,\ x \in S\}.$$

**Proof.** Let $x, y \in S$. Then we have

$\wedge\, \mu_{Ai}(xy) \geq \wedge\, \{min\{\mu_{Ai}(x), \nu_{Ai}(y)\}\}$

$$= min\, \{min\{\mu_{Ai}(x), \mu_{Ai}(y)\}\},$$
$$= min\, \{min\{\mu_{Ai}(x)\}, min\{\mu_{Ai}(y)\}\}$$
$$= min\{\wedge\, \mu_{Ai}(x), \wedge\, \mu_{Ai}(y)\},$$

$\vee\, \nu_{Ai}(xy) \leq \vee\, \{max\{\nu_{Ai}(x), \nu_{Ai}(y)\}\}$

$$= max\, \{max\{\nu_{Ai}(x), \nu_{Ai}(y)\}\}$$
$$= max\, \{max\{\nu_{Ai}(x)\}, max\{\nu_{Ai}(y)\}\}$$
$$= max\{\vee\, \nu_{Ai}(x), \vee\, \nu_{Ai}(y)\}.$$

Hence $\cap A_i$ is an intuitionistic fuzzy sub semigroup of $S$. Next for $x, y, a \in S$ we have

$\wedge\, \mu_{Ai}(x)(xay) \geq \wedge\, \{min\{\mu_{Ai}(x), \mu_{Ai}(y)\}\}$

$$= min\, \{min\{\mu_{Ai}(x), \mu_{Ai}(y)\}\}$$
$$= min\, \{min\{\mu_{Ai}(x)\}, min\{\mu_{Ai}(y)\}\}$$
$$= min\{\wedge\, \mu_{Ai}(x), \wedge\, \mu_{Ai}(y)\},$$

$\vee\, \nu_{Ai}(xay) \leq \vee\, \{max\{\nu_{Ai}(x), \nu_{Ai}(y)\}\}$

$$= max\, \{max\{\nu_{Ai}(x), \nu_{Ai}(y)\}\}$$
$$= max\, \{max\{\nu_{Ai}(x)\}, max\{\nu_{Ai}(y)\}\}$$
$$= max\{\vee\, \nu_{Ai}(x), \vee\, \nu_{Ai}(y)\}.$$

Hence $\cap A_i$ is an intuitionistic fuzzy bi-ideal of $S$.

**Theorem 17** [17] If an IFS $A = \langle \mu_A, \nu_A \rangle$ in $S$ is an intuitionistic fuzzy bi-ideal of $S$ then so is $A = \langle \mu_A, \overline{\mu_A} \rangle = \langle \mu_A, 1 - \mu_A \rangle$.

**Proof**. It is sufficient to show that $\overline{\mu_A}$ satisfies the condition (ii) in Definition (3.5). For any $a, x, y \in S$, we have

□

$$\overline{\mu_A}(xy) = 1 - \mu_A(xy) \leq 1 - min\{\mu_A, \mu_A(y)\}$$
$$= max\{1 - \mu_A(x), 1 - \mu_A(y)\} = max\{\overline{\mu_A}(x), \overline{\mu_A}(y)\}$$

and

$$\overline{\mu_A}(xay) = 1 - \mu_A(xay) \leq 1 - min\{\mu_A(x), \mu_A(y)\}$$
$= max\{1 - \mu_A(x), 1 - \mu_A(y)\} = max\{\overline{\mu_A}(x), \overline{\mu_A}(y)\}$ is an Therefore $A$ is an intuitionistic fuzzy bi-ideal of $S$.

**Definition 18** [17] Let $A$ be a semigroup. An intuitionistic fuzzy set $A = \langle \mu_A, \nu_A \rangle$ is said to be an intuitionistic fuzzy (1,2)-ideal of $S$ if $\forall x, y, z, w \in S$

**(i)** $\mu_A(xw(yz) \geq min\{\mu_A(x), \mu_A(y), \mu_A(z)\}$

**(ii)** $\nu_A(xw(yz) \leq max\{\nu_A(x), \nu_A(y), \nu_A(z)\}$.

**Example 19** Let $S = \{a, b, c, d, e\}$ be a semigroup with the following Cayley table:

| . | | | | | |
|---|---|---|---|---|---|
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |

Define an IFS $A = \langle \mu_A, \nu_A \rangle$ in $S$ by $\mu_A(a) = \mu_A(b) = \mu_A(c) = 1$, $\mu_A(d) = \mu_A(e) = 0$, $\nu_A(a) = \nu_A(b) = \nu_A(c) = 0$, $\nu_A(d) = \nu_A(e) = 1$. By routine calculation, it is clear $A = \langle \mu_A, \nu_A \rangle$ is an intuitionistic fuzzy (1,2)-ideal of $S$.

We note that $M = \{a, b, c\}$ is an a (1,2)-ideal of $S$, hence $A = \langle \mu_A, \nu_A \rangle$ can be defined as follows:

$$\mu A(x) = \begin{cases} 1 & if\ x \in M \\ 0 & otherwise \end{cases} \quad \text{and} \quad \nu A(x) = \begin{cases} 0 & if\ x \in M \\ 1 & otherwise \end{cases}$$

**Theorem 20** [15] Every intuitionistic fuzzy bi-ideal is an intuitionistic fuzzy (1, 2)ideal.

**Proof.** Let $A = \langle \mu_A, \nu_A \rangle$ be an intuitionistic fuzzy bi-ideal of $S$ and let $z, x, y, w \in S$.
Then

$\mu_A(xz(yw)) = \mu_A((xyz)w)$

$$\geq inf\{\mu_A(xyz), \mu_A(w)\}$$
$$\geq inf\{inf\{\mu_A(x), \mu_A(y)\}, \mu_A(w)\}$$
$$= inf\{\{\mu_A(x), \mu_A(y)\}, \mu_A(w)\} \text{ and}$$

$\nu_A(xz(yw)) = \nu_A((xyz)w)$

$$\leq sup\{\nu_A(xyz), \nu_A(w)\}$$
$$\leq sup\{inf\{\nu_A(x), \nu_A(y)\}, \nu_A(w)\}$$
$$= sup\{\{\nu_A(x), \nu_A(y)\}, \nu_A(w)\}.$$

Hence $A = \langle \mu_A, \nu_A \rangle$ is an intuitionistic fuzzy (1, 2)-ideal.

To consider the converse of Theorem (3.20), we need to strengthen of a semgroup $S$.

**Theorem 21** [15] If $S$ is a regular semigroup, then every intuitionistic fuzzy (1, 2)-ideal of $S$ is an intuitionistic fuzzy bi-ideal of $S$.

**Proof**. Assume that a semigroup $S$ is regular and let $A = \langle \mu_A, \nu_A \rangle$ be an intuitionistic fuzzy bi-ideal of $S$ and let $z, x, y \in S$. Since $S$ is regular, we have $xz \in (xSx)S \subseteq xSx$, which implies $xz = xsx$ for some $s \in S$ .Thus

$$\begin{aligned} \mu_A(xzy) &= \mu_A((xsx)y) = \mu_A(xs(xy) \\ &\geq inf\{\mu_A(x), \mu_A, (x), \mu_A(y)\} \\ &= inf\{\mu_A(x), \mu_A(y)\}, \text{ and} \\ \nu_A(xzy) &= \nu_A((xsx)y) = \nu_A(xs(xy) \leq sup\{\nu_A(x), \nu_A(x), \nu_A(y)\} \\ &= sup\{\nu_A(x), \nu(y)\}. \end{aligned}$$

Therefore $A = \langle \mu_A, \nu_A \rangle$ is an intuitionistic fuzzy bi-ideal of $S$.

**Theorem 22 [15]** Let $A = \langle \mu_A, \nu_A \rangle$ be an intuitionistic fuzzy bi-ideal of $S$. If $S$ is a completely regular, then $A(a) = A(a^2)$ for all $a \in S$.

**Proof.** Let $a \in S$. Then there exists $x \in S$ such that $a = a^2xa^2$. Hence

$$\begin{aligned} \mu_A(a) &= \mu_A(a^2xa^2) \geq inf\{\mu_A(a^2), \mu_A(a^2)\} = \mu_A(a^2) \geq inf\{\mu_A(a), \mu_A(a)\} = \mu_A(a) \text{ and} \\ \nu_A(a) &= \nu_A(a^2xa^2) \leq sup\{\nu_A(a^2), \nu_A(a^2)\} \\ &= \nu_A(a^2) \leq sup\{\nu_A(a), \nu_A(a)\} = \nu_A(a) \end{aligned}$$

It follows that $\mu_A(a) = \mu_A(a^2)$ and $\nu_A(a) = \nu_A(a^2)$ so that $A(a) = A(a^2)$ .

**Theorem 23** [15] Let$A = \langle \mu_A, \nu_A \rangle$ be an intuitionistic fuzzy ideal of $S$. If $S$ is an intra- regular, then $A(a) = A(a^2)$ for all $a \in S$.

**Proof.** Let a be any element of $S$. Then since $S$ is intra-regular, there exist $x$ and $y$ in $S$ such that $a = xa^2y$. Hence since $A = \langle \mu_A, \nu_A \rangle$ is an intuitionistic fuzzy ideal,

$$\begin{aligned} \mu_A(a) &= \mu_A(xa^2y) \geq \mu_A(xa^2) \\ &\geq \mu_A(a^2) \geq inf\{\mu_A(a), \mu_A(a)\} = \mu_A(a) \text{ and} \\ \nu_A(a) &= \nu_A(xa^2y) \leq \nu_A(xa^2) \leq \nu_A(a^2) \\ &\leq sup\{\nu_A(a), \nu_A(a)\} = \nu_A(a). \end{aligned}$$

Hence we have $\mu_A(a) = \mu_A(a^2)$ and $\nu_A(a) = \nu_A(a^2)$. Therefore $A(a) = A(a^2)$ for all $xy \in S$.

**Theorem 24** [15] Let $A = \langle \mu_A, \nu_A \rangle$ be an intuitionistic fuzzy ideal of $S$. If $S$ is an intra- regular, then $A(ab) = A(ba)$ for all $a, b \in S$. **Proof.** Let $a, b \in S$. Then by Theorem 3.23, we have

$$\begin{aligned} \mu_A(ab) &= \mu_A((ab)^2) \geq \mu_A(a(ba)b) \\ &\geq \mu_A(ba) = \mu_A((ba)^2) \geq \mu_A(b(ab)a) \geq \mu_A(ab) \text{ and} \\ \nu_A(ab) &= \nu_A((ab)^2) \leq \nu_A(a(ba)b) \leq \nu_A(ba) \\ &= \nu_A((ba)^2) \leq \nu_A(b(ab)a) \leq \nu_A(ab) \ . \end{aligned}$$

So we have $\mu_A(ab) = \mu_A(ba)$ and $\nu_A(ab) = \nu_A(ba)$ .Therefore $A(ab) = A(ba)$ .

**Theorem 25** [15] An IFS $A = \langle \mu_A, \nu_A \rangle$ is an intuitionistic fuzzy bi-ideal of $S$ if and only if the fuzzy sets $\mu_A$ and $\overline{\overline{\nu_A}}$ are fuzzy bi-ideals of $S$.

**Proof.** Let $A = \langle \mu_A, \nu_A \rangle$ be an intuitionistic fuzzy bi-ideal of $S$. Then clearly $\mu_A$ is a fuzzy bi-ideal of $S$. Let $x, a, y \in S$. Then

$$\begin{aligned} \overline{\nu_A}(xy) &= 1 - \nu_A(xy) \\ &\geq 1 - sup\{\nu_A(x), \nu_A(y)\} \\ &= inf\{(1 - \nu_A(x)), (1 - \nu_A(y))\} \\ &= inf\{\overline{\nu_A}(x), \overline{\nu_A}(y)\}, \end{aligned}$$

**And**

$$\begin{aligned} \overline{\nu_A}(xay) &= 1 - \nu_A(xay) \\ &\geq 1 - sup\{\nu_A(x), \nu_A(y)\} \\ &= inf\{(1 - \nu_A(x)), (1 - \nu_A(y))\} \\ &= sup\{\nu_A(x), \nu_A(y)\}, \end{aligned}$$

Conversely, suppose that $\mu_A$ and $\overline{\nu_A}$ are fuzzy bi-ideal of $S$. Let $x, a, y \in S$. Then

$$1 - \nu_A(xy) = \overline{\nu_A}(xy)$$

$$\geq inf\{\overline{\nu_A}(x), \overline{\nu_A}(y)\}$$

$$= 1 - sup\{\nu_A(x), \nu_A(y)\}$$

Which imply that $\nu_A(xy) \leq sup\{\nu_A(x), \nu_A(y)\}$ and $\nu_A(xay) \leq sup\{\nu_A(x), \nu_A(y)\}$.

**Corollary 26 [15]** An IFS $A = \langle \mu_A, \nu_A \rangle$ is an intuitionistic fuzzy bi-ideal of $S$ if and only if $A = \langle \mu_A, \overline{\mu_A} \rangle$ and $\Diamond$ $A = \langle \overline{\nu_A}, \nu_A \rangle$ are intuitionistic fuzzy bi-ideals of $S$.

**Proof.** It is straightforward by Theorem 3.25.

**Theorem 27 [15]** Let $f: S \to T$ be a homomorphism of semigroups. If $B = \langle \mu_B, \nu_B \rangle$ is an intuitionistic fuzzy bi-ideal of $T$, then the preimage $f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\nu_B))$ of $B$ under $f$ is an intuitionistic fuzzy bi-ideal of $S$.

**Proof.** Assume that $B = \langle \mu_B, \nu_B \rangle$ is an intuitionistic fuzzy bi-ideal of $T$ and let $x, y \in S$. Then

$f^{-1}(\mu_B)(xy) = \mu_B(f(xy))$

$= \mu_B(f(x)f(y))$

$\geq min\{\mu_B(f(x)), \mu_B(f(y))\}$

$= min\{(f^{-1}(\mu_B(x)), f^{-1}(\mu_B(y))\}$, and

$f^{-1}(\nu_B)(xy) = \nu_B(f(xy))$

$= \nu_B(f(x)f(y))$

$\leq max\{\nu_B(f(x)), \nu_B(f(y))\}$

$= max\{(f^{-1}(\nu_B(x)), f^{-1}(\nu_B(y))\}$

Hence $f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\nu_B))$ is an intuitionistic fuzzy subsemigroup of $S$. For any $a, x, y \in S$ we have

$f^{-1}(\mu_B)(xay) = \mu_B(f(xay))$

$= \mu_B(f(x)f(a)f(y))$

$\geq min\{\mu_B(f(x)), \mu_B(f(y))\}$

$= min\{(f^{-1}(\mu_B(x), f^{-1}(\mu_B(y))\}$, and

$f^{-1}(\nu_B)(xay) = \nu_B(f(xay))$

$= \nu_B(f(x)f(a)f(y))$

$\leq max\{\nu_B(f(x)), \nu_B(f(y))\}$

$= max\{(f^{-1}(\nu_B(x), f^{-1}(\nu_B(y))\}$.

Hence $f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\nu_B))$ is an intuitionistic fuzzy bi-ideal of $S$.

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