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TREATMENT OF HALF-LIFE VALUES OF SOME LEAD ISOTOPES FOR THE EMISSION OF ALPHA-PARTICLES IN 178≤A≤194

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Abstract:-
To develop equations to calculate the half-life of lead isotopes theoretically and compare them with practical results by assuming a simple mathematical model based on the probability of forming alpha particles inside the parent nucleus before tunneling decay them. It is assumed that the presence of an alpha particle inside the nucleus has a Coulomb effect dependent on the reduced mass of the alpha particle with the core, reducing the rest of the effects on the effective potential. The effective radius depends on the mass number of the core and the cluster. The model assumes that there is a number of collisions before tunneling decay, taking into account the existence of the potential barrier penetration coefficient, which expresses the potential permeability of the potential barrier to the alpha particles within it. The model assumes that the Gamow coefficient is dependent on the atomic numbers of both core and cluster, the reduced mass, and the effect of the energy of the alpha particles and takes into account the value of the Coulomb effect.

Keywords:-*Alpha decay, Half-life, Coulomb potential*

INTRODUCTION

In the beginning, the models that took into account the relationship between alpha decay half-lives and energy of alpha particles for even – even nuclei are presented; Nearly a century ago, Geiger and Nuttall [1] found that the logarithm of the half-life in alpha decay is inversely proportional to the energy of the outgoing alpha particle (*i.e.*, the decay energy Q_α). A straight line was obtained by plotting the logarithm of the half-life versus $\frac{1}{\sqrt{Q_\alpha}}$ for groundstate transitions of even – even nuclei. Viola and Seaborg [2] generalized the Geiger – Nuttall law with additional adjustable parameters and proposed a simple formula for alpha decay halflives of heavy nuclei. Parkhomenko and Sobiczewski [3] presented a new version of the Viola–Seaborg formula for heavy and super heavy nuclei. Recently, the renewed interest in alpha decay systematic has been stimulated by the unified description of various hadonic decays. A unified law (New Geiger–Nuttall law) of alpha decay was proposed [4, 5]. Heavy nuclei tend to be unstable because of the Coulomb repulsion of the large number of protons carried by it. As the alpha particle is highly stable and tightly bound structure, hence, it becomes the natural choice of these heavy nuclei to get rid of some of its extra positive charges through alpha decay [6], we phenomenological formula provided relationship connecting the half-life and the energy of the α-particles. A-decay studies have been reliably used for providing valuable information about the ground state half-life, ground state energy, nuclear spin and parity, shell effects, α-decay plays a significant role in the study of nuclear structure and in the synthesis of new super-heavy elements. The Coulomb interaction between the two nuclei with is the first term taken from Ref.[7], the interaction potential between the two nuclei can be written as the Coulomb potential, we focused only on the coulomb potential to show how important it is in interpreting nuclei disintegration process by alpha particles(the simple theory of α-decay). What is new in this work is that there is a significant difference between practical and theoretical results [8], and this will be treated with a theoretical model to reduce this difference between practical and theoretical results.

1. Results and Discussion

The nuclear system consisting of the core nucleus and an alpha nucleus (Cluster) is actually an asymmetric nuclear molecule [9]. Both nuclei are in the ground state and interact with each other through the nucleus-nucleus potential [10,11]. Condensation of valence nucleons of the parent nucleus to a Cluster is accompanied by energy release, Q ≈ tens of energy in MeV. The amount of energy Q is converted to the oscillatory-motion of the light Nucleus-Cluster inside the parent nucleus. Generally, Q is always much smaller than the exit Coulomb potential barrier, V(r),of the parent nucleus.The process of Cluster emission is a result of quantum-mechanical penetration through the V(r) of the parent nucleus [12].

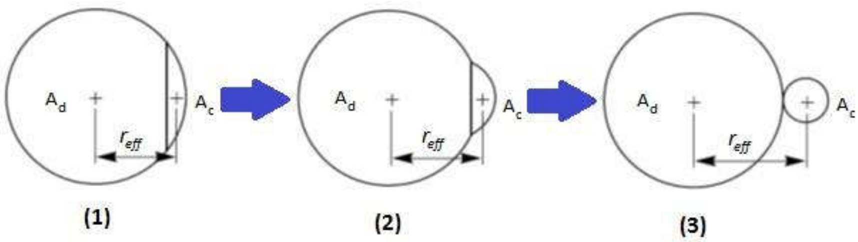

Fig. 1

Alpha radioactivity proceeds in two stages, the first stage;formation of an asymmetric nuclear molecule (Nucleus-Cluster) then the second stage is the emission of the formed Cluster (alpha decay process) [13]. The theoretical approaches to the description of alpha radioactivity include consideration of both stages. The main difference between them is the description of the alpha formation mechanism. In the adiabatic approach the nuclear molecule results from super asymmetric deformation of the initial heavy nucleus. Figure 1 taken from demonstrates an example of how this stage proceeds. In the non-adiabatic approach the nuclear molecule results from quantum fluctuation or quantum fragmentation in the initial state of the heavy nucleus. As a result of the repeated collision of alpha particles with the potential barrier, it is possible to penetrate it (see Appendix 1)

$$\lambda_\alpha = P_\alpha f_\alpha T \quad \dots (1)$$

P_α defined as the probability of formation of the α-particle as a separate cluster inside the parentnucleus before the emission process, the range of values should be, $0 < P_\alpha \leq 1$, In our search we took a value of P_α equal to one.

The number of collisions per second carried out by the alpha with the potential barrier depends on its velocity, which can be found from the energy of the movement of the alpha particles, taking into account the value of the reduced mass of the system, depending on the mass numbers of the core and the cluster, At maximum speed, the total energy of the system is the sum of the two potential energy and the alpha particle decay energy, allowing the calculation of the velocity of the alpha particles (see Appendix 1). In range is equal to twice the effective radius between them, and the frequency is:

$$f_\alpha = \frac{\sqrt{\frac{2(Q_\alpha + V(r))C^2}{\mu}}}{2r_{eff}} \quad \dots (2)$$

The interaction potential between the two nuclei can be written as the Coulomb potential, we focused only on the coulomb potential to show how important it is in interpreting nuclei disintegration process by alpha particles (the simple theory of α-decay). The penetration of the alpha particles to the potential barrier depends on the ratio between the intensity of the current wave and the intensity of the falling wave, showing that the probability at the edge of the potential barrier decreases exponentially with increasing distance (see Appendix 1)

$$T_\alpha = e^{-2G} \dots (3)$$

But

$$t_{\frac{1}{2}} = \frac{ln2}{\lambda_\alpha} \dots (4)$$

We arrive at the proposed final picture of the linear relationship between the half-lives of the matrilineal nuclei of the nuclei - the maturation of the root of the dissolution of the energy of the alpha particles, (See Appendix 1)

Where

$$log t_{\frac{1}{2}} = C_1 \sqrt{\frac{1}{Q_\alpha}} + C_2 \dots (5)$$

$$C_1 = Z_c Z_d \pi e^2 \sqrt{\frac{2\mu}{\hbar^2}} log[e] \dots (6)$$

$$C_2 = log\left[\frac{2r_{eff} ln2}{\sqrt{\frac{2V(r)c^2}{\mu}}}\right] - 4Z_c Z_d e^2 \sqrt{\frac{2\mu}{\hbar^2 V(r)}} log[e] \dots (7)$$

It is similar in style to the famous Geiger and Nuttall formula. In this work we consider the alternate approach and present direct calculations of the half-life of the heavy nuclei. The nuclei considered are the even-even isotopes of all known α-emitters, starting from A_p = 178 up to A_p = 194, and Z_p = 82.

Heavy nuclei Pb (Z = 82) are unstable because of the Coulomb repulsion of the large number of protons inside it. Since the α-particle is highly stable and tightly bound structure, for the reasons, it becomes the natural choice of these heavy nuclei to get rid of some of its extra positive charges through α-decay. We phenomenological formula provided relationship connecting the half-life and the energy of the α-particles. α-decay studies have been reliably used for providing valuable information about the ground state half-life, ground state energy, nuclear spin and parity, shell effects, α-decay plays a significant role in the study of nuclear structure and in the synthesis of new super heavy elements. The comparison of our calculations and the experimental data [14-22] for typical nuclei is given in table (1). Comparing the predicted half-life given by our formula model, for nuclei $^{178-194}$Pb, with the corresponding experimental data, one notices that the results of our formula model are good agreement with experimental data. Therefore, the studies indicate that, the formula model may provide an accurate description for the considered nuclei.

Table (1). Comparison of the calculated logarithms partial α-decay half-lives for even–even Pb nuclei with the available experimental data. The frequency are also shown for nuclei.

Isotopes / Properties	^{178}Pd	^{180}Pd	^{182}Pd	^{184}Pd	^{186}Pd	^{188}Pd	^{190}Pd	^{192}Pd	^{194}Pd
A	178	180	182	184	186	188	190	192	194
N	96	98	100	102	104	106	108	110	112
Q (MeV)	7.824	7.452	7.099	6.807	6.504	6.143	5.732	5.255	4.772
Fx10^{21}(sec-1)	2.67	2.74	2.72	2.71	2.71	2.69	2.67	2.63	2.68
LogT$_{1/2}$(Theo.) (sec)	-3.924	-2.384	-1.246	-0.218	1.002	2.433	4.253	6.544	9.782
LogT$_{1/2}$(Exper.) (sec)	-3.921	-2.377	-1.251	-0.213	1.083	2.431	4.250	6.551	9.945

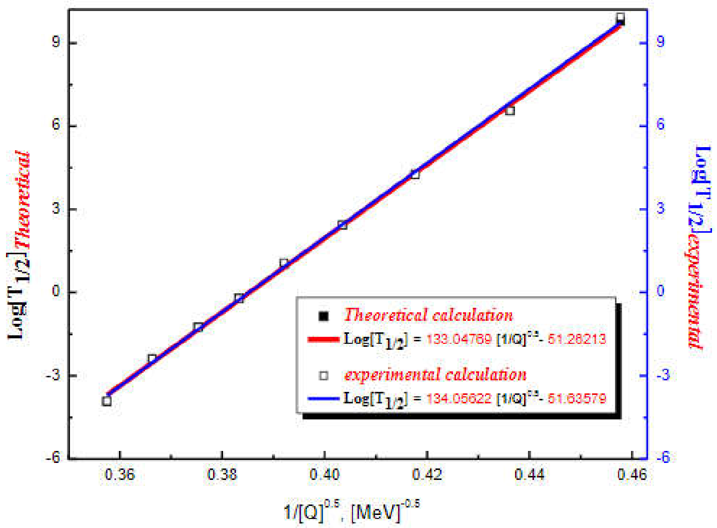

Fig. 1. The logarithms of experimental and theoretical partial α-decay half-lives (in s) for the even–even Pb nuclei with neutron number 96 < N < 112 as a function of Q $^{-1/2}$ (in MeV$^{-1/2}$). The straight lines show the description of the formula law with C$_1$ and C$_2$ values fitted for nuclei.

Figure 1 shows the results obtained with the formula of Eq. (5). By comparing it with Table 1, one can see that although a smaller number of adjustable parameters (two) used by the formula, it suitable describes measured half-lives $\log T_{\frac{1}{2}}$. Comparing theoretical with experimental data, one can observe that the quality of description of $\log T_{\frac{1}{2}}$ by the formula is quite similar to that of the GN formula which uses two fitted parameters.

3. Conclusion

we have evaluated the alpha decay half-lives of Pb (Z = 82) isotopes in the region 178 ≤ ≤ 194 within the Coulomb Potential Model (CPM) of our formula, for all the transitions with minimum angular momentum transfer (l = 0), have analyzed the behavior of these isotopes against alpha decay and thus have verified the role of neutron shell closure N = 126 on the alpha decay half-lives. We obtain approximately the half-lives and the frequencies.

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Indexes

Theoretical Description of α −Particles Emission alpha particles emission relies on calculating the rate of emission rate of emission = decay constant $= \lambda_\alpha$ decay constant is product of frequency factor and transmission coefficient

$$\lambda_\alpha = f_\alpha T_\alpha \ \dots (1)$$

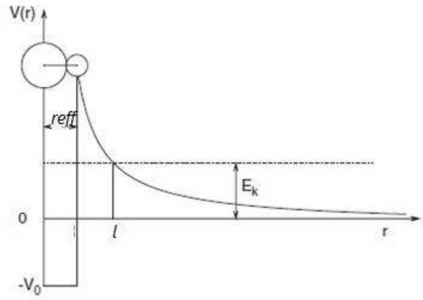
But, additional factor should be multiplied by equation(1), is probability of preformation of α −particles inside the parent nucleus (P_α).P_α defined as the probability of formation of the α −particle as a separate cluster inside the parent nucleus before the emission process the range of values should be, $0 < P_\alpha \leq 1$.

$$\lambda_\alpha = P_\alpha f_\alpha T \ \dots (2)$$

In our search we took a value of P_α equal to one.

$$\lambda_\alpha = f_\alpha T \ \dots (3)$$

Frequency (The assault.1 frequency of the α-particle at the barrier) :



Alpha particle (cluster) is moving inside unstable nucleus distance is twice distance between two centers of cluster and (core) residual nucleons of parent nucleus distance = $2r_{eff}$ … (4)

$$r_{eff} = r_0\left(A_c^{\frac{1}{3}} + A_d^{\frac{1}{3}}\right) \ \dots (5)$$

Where, r_0 is ideal radius, A_c, A_d are mass numbers of cluster and core nuclei, respectively.
Alpha particle (cluster) have kinetic energy

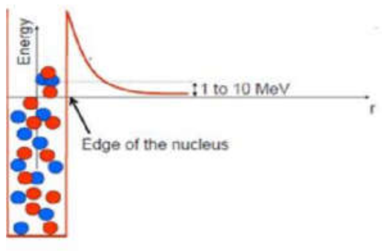
$$KE_\alpha = \frac{1}{2}\mu v_\alpha^2 \ \dots (6)$$

Where, μ is reduced mass of cluster and core nuclei

$$\mu = \frac{M_c M_d}{M_c + M_d} \ \dots (7)$$

$$v_\alpha^2 = \frac{2KE_\alpha}{\mu} \ \dots (8)$$

$$v_\alpha = \sqrt{\frac{2KE_\alpha}{\mu}} \ \dots (9)$$



At maximum velocity of cluster nucleus, At v_α^{max}

$$KE_\alpha = E \ \dots (10)$$

$$E_T = Q_\alpha + (r) \ \dots (11)$$

Where, Q_α, $V(r)$ are kinetic energy of cluster and potential well respectively

$$KE_\alpha = Q_\alpha + V(r) \quad ...(12)$$

$$v_\alpha^{max} = \sqrt{\frac{2(Q_\alpha + V(r))}{\mu}} \quad ...(13)$$

Time to cross unstable nucleus (parent nucleus) is t_α

$$t_\alpha = \frac{2r_{eff}}{v_\alpha^{max}} \quad ...(14)$$

But, how many times the α-particle (cluster) hit barrier per second or knocking rate? The answer to this question is to find frequency

$$f_\alpha = \frac{1}{t_\alpha} \quad ...(15)$$

$$f_\alpha = \frac{v_\alpha^{max}}{2r_{eff}} \quad ...(16)$$

$$f_\alpha = \frac{\sqrt{\frac{2(Q_\alpha + V(r))C^2}{\mu}}}{2r_{eff}} \quad ...(17)$$

Transmission Coefficient: .2

T_α defined as the current density (flux) in region III of incident wave to the current density (flux) in region I of incident wave. 2-1. Time – independent Schrodinger Equation:

$$-\frac{\hbar^2}{2}\nabla^2\Psi + V\Psi = E\Psi \quad ...(18)_\mu$$

Where $E = E_T, V = (r) r$ distance between cluster and core, and $\nabla^2 = \frac{d^2}{dr^2}$ (in one dimension r), $\Psi = \Psi(r)$

$$-\frac{\hbar}{2\mu}\nabla^2(r)\Psi(r) + V(r)\Psi(r) = E_T\Psi(r) \quad ...(19)$$

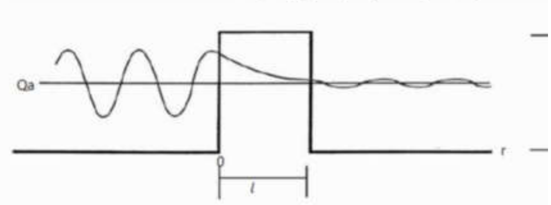
$$-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2}\Psi(r) + V(r)\Psi(r) = E_T\Psi(r) \quad ...(20)$$

Multiplied by equation(20) $-\frac{2\mu}{\hbar^2}$

$$\frac{d^2}{dr^2}\Psi(r) - \frac{2\mu}{\hbar^2}V(r)\Psi(r) = -\frac{2\mu}{\hbar^2}E_T\Psi(r) \quad ...(21)$$

$$\frac{d^2}{dr^2}\Psi(r) = \frac{2\mu}{\hbar^2}V(r)\Psi(r) - \frac{2\mu}{\hbar^2}E_T\Psi(r) \quad ...(22)$$

$$\frac{d^2}{dr^2}\Psi(r) = -\frac{2\mu}{\hbar^2}[E_T - V(r)]\Psi(r) \quad ...(23)$$



Divide space into three regions 2-1.a. In region I:

Let

$\frac{2\mu}{\hbar^2}[E_T - V(r)] = k^2, \quad k$ is wave

Number

$$k = \sqrt{\frac{2\mu}{\hbar^2}[E_T - V(r)]} \;\ldots (24)$$

$$E_T > V(r) \;\; i.e. \; E_T > 0$$

k *Real*

$$\frac{d^2}{dr^2}\Psi_I(r) = -k^2\Psi_I(r) \;\ldots (25)$$

$$\frac{d^2}{dr^2}\Psi_I(r) + k^2\Psi_I(r) = 0 \;\ldots (26)$$

Multiplied by equation (26) i^2, $i^2 = -1$ $d2$

$$i^2\frac{d^2}{dr^2}\Psi_I(r) + i^2k^2\Psi_I(r) = 0 \;\ldots (27)$$

$$-\frac{d^2}{dr^2}\Psi_I(r) + i^2k^2\Psi_I(r) = 0 \;\ldots (28)$$

Multiplied by equation (28) -1

$$\frac{d^2}{dr^2}\Psi_I(r) - i^2k^2\Psi_I(r) = 0 \;\ldots (29)$$

$$\left[\frac{d^2}{dr^2} - i^2k^2\right]\Psi_I(r) = 0 \;\ldots (30)$$

$$\left[\left(\frac{d}{dr} - ik\right)\left(\frac{d}{dr} + ik\right)\right]\Psi_I(r) = 0 \;\ldots (31)$$

Either

$$\left(\frac{d}{dr} - ik\right)\Psi_I(r) = 0 \;\ldots (32)$$

$$\frac{d}{dr}\Psi_I(r) = ik\Psi_I(r) \ldots (33)$$

Separation of variables

$$\frac{d\Psi_I(r)}{\Psi_I(r)} = ikdr \ldots (34)$$

Integration

$$\int \frac{d\Psi_I(r)}{\Psi_I(r)} = \int ikdr \ldots (35)$$

$$\int \frac{d\Psi_I(r)}{\Psi_I(r)} = ik\int dr \ldots (36)$$

$$\int \frac{d\Psi_I(r)}{\Psi_I(r)} = ik\int dr \ldots (37)$$

$$ln\,\Psi(r) = ikr + C_1 \ldots (38)$$

Where C_1 is constant of integration

$$\Psi_I(r) = e^{ikr+C_1} \ldots (39)$$

$$\Psi_I(r) = e^{C_1}e^{ikr} \ldots (40)$$

$$\Psi_I(r) = a_I e^{ikr} \ldots (41)$$

$$\Psi_{I\rightarrow}(r) = \text{incident wave} \ldots \quad \textbf{(42)}$$

Or

$$\left(\frac{d}{dr} + ik\right)\Psi_I(r) = 0 \;\ldots (43)$$

Separation of variables

$$\frac{d}{dr}\Psi_I(r) = -ik\Psi_I(r) \ldots (44)$$

Integration

$$\frac{d\Psi_I(r)}{\Psi_I(r)} = -ikdr \ldots (45)$$

$$\int \frac{d\Psi_I(r)}{\Psi_I(r)} = -\int ikdr \ldots (46)$$
$$\int \frac{d\Psi_I(r)}{\Psi_I(r)} = -ik\int dr \ldots (47)$$
$$\int \frac{d\Psi_I(r)}{\Psi_I(r)} = -ik\int dr \ldots (48)$$
$$ln\ \Psi(r) = -ikr + C_2 \ldots \textbf{(49)}$$

Where C_2 is constant of integration

$$\Psi(r) = e-ikr+C2 \ldots \textbf{(50)}$$
$$\Psi(r) = eC2e-ikr \ldots \textbf{(51)}$$
$$\Psi(r) = b_I e^{-ikr} \ldots \textbf{(52)}$$
$$\Psi_{I\leftarrow}(r) = \text{reflection wave} \ldots (53)$$

The solution of equation (26)

$$\Psi(r) = \Psi_{I\rightarrow}(r) + \Psi_{I\leftarrow}(r) \ldots (54)$$
$$\Psi(r) = a\ eikr+bIe-ikr \ldots (55)\ \Psi_I(r)$$

I

Is traveling wave a_I, b_I are complex coefficients 2-1.b. In region II:

E_T $<(r)$ i.e. $E_T < 0$ k imaginary

$$k_1 = ik \quad \ldots (56)$$
$$k_1 = i\sqrt{\frac{2\mu}{\hbar^2}[E_T - V(r)]} \quad \ldots (57)$$
$$k_1 = \sqrt{\frac{2\mu}{\hbar^2}[V(r) - E_T]} \quad \ldots (58)$$

Equation (26) become

$$\frac{d^2}{dr^2}\Psi_{II}(r) + {k_1}^2\Psi_{II}(r) = 0 \quad \ldots (59)$$
$$k = \frac{k_1}{i} \quad \ldots (60)$$
$$k = \frac{k_1}{i}\frac{i}{i} \quad \ldots (61)$$
$$k = -ik_1 \quad \ldots (62)$$

the solution of equation (59)

$\Psi_{II}(r)$

$$= a_{II}e(-ik^1)r+b_{II}e-i(-ik^1)r \ldots (63)$$
$$\Psi I(r) = aIIek1r+bIIe-k1r \ldots (64)$$

$\Psi_I(r)$ is standing wave

a_{II}, b_I are complex coefficients 2-1.c. In region III:

E $T>(r)$ i.e. $E_T > 0$

E k real

Equation (26) become

$$\frac{d^2}{dr^2}\Psi_{III}(r) + k^2\Psi_{III}(r) = 0 \quad \ldots (65)$$

the solution of equation (65)

$$\Psi_{II}(r) = \Psi_{III\rightarrow}(r) \ldots (66)$$
$$\Psi III(r) = a_{III}e^{ikr} \ldots (67)$$

$\Psi_{II}(r)$ is traveling wave

a_{II} is complex coefficient 2-2. Probability Density:

$|\Psi|^2$ defined as number of particles traversing unit area perpendicular to $\overrightarrow{v_\alpha}$ (in any direction) in unit time, $|\Psi\Psi^*|$

2-2.a. In region I:

probability density of incident wave defined as

$$|\Psi_{I\rightarrow}(r)|^2 = |\Psi_{I\rightarrow}(r)\Psi^*_{I\rightarrow}(r)| \quad \ldots (68)$$
$$|\Psi I\rightarrow(r)|2 = |aIeikraIe-ikr| \ldots (69)$$
$$|\Psi_{I\rightarrow}(r)|^2 = |a_I{}^2e^0| \ldots (70)$$
$$|\Psi_{I\rightarrow}(r)|^2 = |a_I{}^2| \ldots (71)$$

2-2.b. In region III:

probability density of incident wave defined as

$|\Psi III\rightarrow(r)|2$

$$= |\Psi_{III\rightarrow}(r)\Psi_{III^*\rightarrow}(r)| \quad \ldots (72)$$

$$|\Psi III\rightarrow(r)|2$$

$$= |aIIIeikraIIIe{-}ikr| \ldots (73)$$

$$|\Psi III\rightarrow(r)|2 = |aIII2\ e0| \quad \ldots (74)$$

$$|\Psi_{III\rightarrow}(r)|^2 = |a_{III}{}^2| \quad \ldots (75)$$

2.3. Particle Current Density (Particle Flux):

S defined as $S= \frac{\hbar}{2i\mu}\left[\Psi^*(r)\frac{d\Psi(r)}{dr} - \Psi(r)\frac{d\Psi^*(r)}{dr}\right]$

2-3.a. In region III:

From equation (67)

$$\frac{d\Psi_{\mathrm{III}}(r)}{dr} = ika_{III}e^{ikr} \quad \ldots (76)$$

$$\Psi^*_{III}(r) = a_{III}e^{-ikr} \quad \ldots (77)$$

$$\frac{d\Psi^*_{III}(r)}{dr} = -ika_{III}e^{-ikr} \quad \ldots (78)$$

transmitted probability particle current density is

$$S_{\mathrm{transmitted}} = \frac{\hbar}{2i\mu}\left[\Psi^*_{III}(r)\frac{d\Psi_{\mathrm{III}}(r)}{dr} - \Psi_{\mathrm{III}}(r)\frac{d\Psi^*_{III}(r)}{dr}\right] \quad \ldots (79)$$

$$S_{\mathrm{transmitted}} = \frac{\hbar}{2i\mu}\left[a_{III}e^{-ikr}\left(ika_{III}e^{ikr}\right) - a_{III}e^{ikr}\left(-ika_{III}e^{-ikr}\right)\right] \quad \ldots (80)$$

$$S_{\mathrm{transmitted}} = \frac{\hbar}{2i\mu}\left[ika^2_{III} + ika^2_{III}\right] \quad \ldots (81)$$

$$S_{\mathrm{transmitted}} = \frac{2ik\hbar}{2i\mu}a^2_{III} \quad \ldots (82)$$

$$S_{\mathrm{transmitted}} = \frac{k\hbar}{\mu}a^2_{III} \quad \ldots (83)$$

But, momentum is $\mu v_\alpha = k\hbar$

μv

$$S_{\mathrm{transmitted}} = \frac{\alpha}{\mu}a^2_{III} \quad \ldots (84)$$

$$S_{\mathrm{transmitted}} = v_\alpha a^2_{III} \quad \ldots (85)$$

2-3.b. In region I:

From equation (41)

$$\frac{d\Psi_{\mathrm{I}\rightarrow}(r)}{dr} = ika_I e^{ikr} \quad \ldots (86)$$

$$\Psi^*_{I\rightarrow}(r) = a_I e^{-ikr} \quad \ldots (87)$$

$$\frac{d\Psi^*_{I\rightarrow}(r)}{dr} = -ika_I e^{-ikr} \quad \ldots (88)$$

Incident probability particle current density is

$$S_{\text{incident}} = \frac{\hbar}{2i\mu}\left[\Psi^*_{I\rightarrow}(r)\frac{d\Psi_{\mathrm{I}\rightarrow}(r)}{dr} - \Psi_{\mathrm{I}\rightarrow}(r)\frac{d\Psi^*_{I\rightarrow}(r)}{dr}\right] \quad \dots (89)$$

$$S_{\text{incident}} = \frac{\hbar}{2i\mu}\left[a_I e^{-ikr}\left(ika_I e^{ikr}\right) - a_I e^{ikr}\left(-ika_I e^{-ikr}\right)\right] \quad \dots (90)$$

$$S_{\text{tincident}} = \frac{\hbar}{2i\mu}\left[ik\, a_I^{\ 2} + ika_I^2\right] \quad \dots (91)$$

$$S_{\text{incident}} = \frac{2ik\hbar}{2i\mu} a_I^2 \quad \dots (92)$$

$$S_{\text{incident}} = \frac{k\hbar}{\mu} a_I^2 \quad \dots (93)$$

$$S_{\text{incident}} = \frac{\mu v_\alpha}{\mu} a_I^2 \quad \dots (94)$$

$$S_{\text{incident}} = v_\alpha a_I^2 \quad \dots (95)$$

From equation (85) and (95)

$$T_\alpha = \frac{S_{\text{transmitted}}}{S_{\text{incident}}} \quad \dots (96)$$

$$T_\alpha = \frac{v_\alpha a_{III}^2}{v_\alpha a_I^2} \quad \dots (97)$$

$$T_\alpha = \frac{a_{III}^2}{a_I^2} \quad \dots (98)$$

2.4. Determination Complex Coefficients a_I, a_{III}:

Rule: Ψ and $\frac{d}{dr}\Psi$ are continuous at $r = 0$ and $r = l$, where l thickness of barrier.

2.4.a. At $r = 0$:

Boundary barrier

$$\Psi(r = 0) = \Psi_{II}(r = 0) \quad \dots (99)$$

$$\Psi_{\mathrm{I}\rightarrow}(r = 0) + \Psi_{\mathrm{I}\leftarrow}(r = 0) = \Psi_I(r = 0) \quad \dots (100)$$

$$aIe0+bIe0 = aIIe0+bIIe0 \quad \dots (101)$$

$$a_I+b_I = a_{II}+b_I \quad \dots (102)$$

Derivatives

$$\frac{d\Psi_I(r=0)}{dr} = \frac{d\Psi_{II}(r=0)}{dr} \quad \dots (103)$$

$$\frac{d\Psi_{I\rightarrow}(r=0)}{dr} + \frac{d\Psi_{I\leftarrow}(r=0)}{dr} = \frac{d\Psi_{II}(r=0)}{dr} \quad \dots (104)$$

$$ika_I e^0 - ikb_I e^0 = -k_1 a_{II} e^0 + k_1 b_{II} e^0 \quad \dots (105)$$

$$ika - ikb = -k\ a + k\ b \quad \dots\ 106I$$

2.4.b. At $r = l$:

Boundary barrier

$$\Psi_I(r = l) = \Psi_{III}(r = l) \quad \dots (107)$$

$$\Psi_I(r = l) = \Psi_{III\rightarrow}(r = l) \quad \dots (108)$$

$$aIIe{-}k1l+bIIek1l = aIIIeik \quad \dots (109)$$

Derivatives

$$\frac{d\Psi_{II}(r=l)}{dr} = \frac{d\Psi_{III\rightarrow}(r=l)}{dr} \quad \dots (110)$$

$$-k_1 a_{II} e^{-k_1 l} + k_1 b_{II} e^{k_1 l} = k_1 a_{II} e^{ik_1 l} \quad \dots (111)$$

These equations (102), (106), (109) and (111) contain five amplitudes system of linear these equations

$$a_I + b_I - a_{II} - b_{II} + 0a_{III} = 0 \dots (112)$$

$$ika_I - ikb_I + k_1 a_{II} - k_1 b_{II} + 0a_{II} = 0 \dots (113)$$

$$0a_I + 0b_I + e^{-k_1 l} a_{II} + e^{k_1 l} b_{II} - e^{ikl} a_{II} = 0 \dots (114)$$

$$0a_I + 0b_I - k_1 e^{-k_1 l} a_{II} + k_1 e^{k_1 l} b_{II} - ike^{ikl} a_{III} = 0 \dots (115)$$

Solving this system by forward eliminations coefficients matrix

$$\begin{bmatrix} 1 & 1 & -1 & -1 & 0 \\ ik & -ik & k_1 & -k_1 & 0 \\ 0 & 0 & e^{-k_1 l} & e^{k_1 l} & -e^{ikl} \\ 0 & 0 & -k_1 e^{-k_1 l} & k_1 e^{k_1 l} & -ike^{ikl} \end{bmatrix} \dots (116)$$

multiplied by $(-ik)R_1 + R_2$ in matrix

$$\begin{bmatrix} 1 & 1 & -1 & -1 & 0 \\ 0 & -2ik & (k_1 + ik) & (ik - k_1) & 0 \\ 0 & 0 & e^{-k_1 l} & e^{k_1 l} & -e^{ikl} \\ 0 & 0 & -k_1 e^{-k_1 l} & k_1 e^{k_1 l} & -ike^{ikl} \end{bmatrix} \dots (117)$$

multiplied by $k_1 R_3 + R_4$ in matrix

$$\begin{bmatrix} 1 & 1 & -1 & -1 & 0 \\ 0 & -2ik & (k_1 + ik) & (ik - k_1) & 0 \\ 0 & 0 & e^{-k_1 l} & e^{k_1 l} & -e^{ikl} \\ 0 & 0 & 0 & 2k_1 e^{k_1 l} & -(k_1 + ik)e^{ikl} \end{bmatrix} \dots (118)$$

In the following can be written system

$$a_I + b_I - a_{II} - b_{II} + 0a_{III} = 0 \quad \dots (119)$$

$$0a_I - 2ikb_I + (k_1 + ik)a_{II} + (ik - k_1)b_{II} + 0a_{III} = 0 \quad \dots (120)$$

$$0a_I + 0b_I + e^{-k_1 l} a_{II} + e^{k_1 l} b_{II} - e^{ikl} a_{III} = 0 \quad \dots (121)$$

$$0a_I + 0b_I + 0a_{II} + 2k_1 e^{k_1 l} b_{II} - (k_1 + ik)e^{ikl} a_{III} = 0 \quad \dots (122)$$

Number of equations less than number of unknowns

There are infinite number of solutions suppose new variable let

$$a_{III} = \tau \quad \dots (123)$$

From equation (122)

$$2k_1 e^{k_1 l} b_{II} - (k_1 + ik)e^{ikl}\tau = 0 \quad \dots (124)$$

$$2k_1 e^{k_1 l} b_{II} = (k_1 + ik)e^{ikl}\tau \quad \dots (125)$$

$$b_{II} = \frac{(k_1 + ik)}{2k_1} \frac{e^{ikl}}{e^{k_1 l}} \tau \quad \dots (126)$$

$$b_{II} = \frac{k_1 + ik}{2k_1} e^{(ik-k_1)l}\tau \quad \dots (127)$$

From equation (121)

$$e^{-k_1 l} a_{II} + e^{k_1 l}\left[\frac{(k_1 + ik)}{2k_1} e^{(ik-k_1)l}\tau\right] - e^{ikl}\tau = 0 \quad \dots (128)$$

$$a_{II} + e^{2k_1 l}\left[\frac{(k_1 + ik)}{2k_1} e^{(ik-k_1)l}\tau\right] - e^{(ik+k_1)l}\tau = 0 \quad \dots (129)$$

$$a_{II} + \left[\frac{(k_1 + ik)}{2k_1} e^{(ik+k_1)l}\tau\right] - e^{(ik+k_1)l}\tau = 0 \quad \dots (130)$$

$$a_{II} + \left[\frac{(k_1 + ik)}{2k_1} - 1\right] e^{(ik+k_1)l}\tau = 0 \quad \dots (131)$$

$$a_{II} + \left[\frac{(k_1 + ik)}{2k_1} - \frac{2k_1}{2k_1}\right] e^{(ik+k_1)l}\tau = 0 \quad \dots (132)$$

$$a_{II} + \left[\frac{k_1 + ik - 2k_1}{2k_1}\right] e^{(ik+k_1)l}\tau = 0 \quad \dots (133)$$

$$a_{II} + \left[\frac{ik - k_1}{2k_1}\right] e^{(ik+k_1)l}\tau = 0 \quad \dots (134)$$

$$a_{II} = \left[\frac{k_1 - ik}{2k_1}\right] e^{(ik+k_1)l}\tau \quad \dots (135)$$

From equation (120)

$$-2ikb_I + \left\{\frac{(k_1 - ik)(k_1 + ik)}{2k_1} e^{(ik+k_1)l}\tau\right\} + \left[\frac{(ik - k_1)(ik + k_1)}{2k_1} e^{(ik-k_1)l}\tau\right] = 0 \quad \dots (137)$$

$$b_I - \left[\frac{k_1^2 + k^2}{4ikk_1} e^{(ik+k_1)l}\tau\right] - \left[\frac{-k_1^2 - k^2}{4ikk_1} e^{(ik-k_1)l}\tau\right] = 0 \quad \dots (138)$$

$$b_I - \left[\frac{k_1^2+k^2}{4ikk_1}e^{(ik+k_1)l}\tau\right] + \left[\frac{k_1^2+k^2}{4ikk_1}e^{(ik-k_1)l}\tau\right] = 0 \quad \dots(139)$$

$$b_I = \left[\frac{k_1^2+k^2}{4ikk_1}e^{(ik+k_1)l}\tau\right] - \left[\frac{k_1^2+k^2}{4ikk_1}e^{(ik-k_1)l}\tau\right] \quad \dots(140)$$

From equation(119)

$$a_I = a_{II} + b_{II} - b_I \quad \dots(141)$$

$$a_I = \frac{k_1-ik}{2k_1}e^{(ik+k_1)l}\tau + \frac{k_1+ik}{2k_1}e^{(ik-k_1)l}\tau - \frac{k_1^2+k^2}{4ikk_1}e^{(ik+k_1)l}\tau + \frac{k_1^2+k^2}{4ikk_1}e^{(ik-k_1)l}\tau \quad \dots(142)$$

$$a_I = \left[\frac{k_1-ik}{2k_1} - \frac{k_1^2+k^2}{4ikk_1}\right]e^{(ik+k_1)l}\tau + \left[\frac{k_1+ik}{2k_1} + \frac{k_1^2+k^2}{4ikk_1}\right]e^{(ik-k_1)l}\tau \quad \dots(143)$$

$$a_I = \left[\frac{2ikk_1+2k^2-(k_1^2+k^2)}{4ikk_1}\right]e^{(ik+k_1)l}\tau + \left[\frac{2ikk_1-2k^2+k_1^2+k^2}{4ikk_1}\right]e^{(ik-k_1)l}\tau \quad \dots \quad \dots\dots \quad \textbf{(144)}$$

$$a_I = \left\{\left[\frac{2ikk_1+2k^2-k_1^2-k^2}{4ikk_1}\right]e^{(ik+k_1)l} + \left[\frac{2ikk_1-2k^2+k_1^2+k^2}{4ikk_1}\right]e^{(ik-k_1)l}\right\}\tau \quad \dots \quad \dots\dots \quad \textbf{(145)}$$

$$a_I = \left\{\left[\frac{k^2-k_1^2+2ikk_1}{4ikk_1}\right]e^{(ik+k_1)l} + \left[\frac{-k^2+k_1^2+2ikk_1}{4ikk_1}\right]e^{(ik-k_1)l}\right\}\tau \quad \dots(146)$$

$$a_I = \left\{\left[\frac{2ikk_1}{4ikk_1} + \frac{k^2 - k_1^2}{4ikk_1}\right] e^{(ik+k_1)l} + \left[\frac{2ikk_1}{4ikk_1} + \frac{k_1^2 - k^2}{4ikk_1}\right] e^{(ik-k_1)l}\right\}\tau \quad \ldots(147)$$

$$a_I = \left\{\left[\frac{1}{2} + \frac{1}{4i}\left(\frac{k^2}{kk_1} - \frac{k_1^2}{kk_1}\right)\right] e^{(ik+k_1)l} + \left[\frac{1}{2} + \frac{1}{4i}\left(\frac{k_1^2}{kk_1} - \frac{k^2}{kk_1}\right)\right] e^{(ik-k_1)l}\right\}\tau \quad \ldots(148)$$

$$a_I = \left\{\left[\frac{1}{2} + \frac{1}{4i}\left(\frac{k}{k_1} - \frac{k_1}{k}\right)\right] e^{(ik+k_1)l} + \left[\frac{1}{2} + \frac{1}{4i}\left(\frac{k_1}{k} - \frac{k}{k_1}\right)\right] e^{(ik-k_1)l}\right\}\tau \quad \ldots(149)$$

$$a_I = \left\{\left[\frac{1}{2} + \frac{i}{4i^2}\left(\frac{k}{k_1} - \frac{k_1}{k}\right)\right] e^{(ik+k_1)l} + \left[\frac{1}{2} + \frac{i}{4i^2}\left(\frac{k_1}{k} - \frac{k}{k_1}\right)\right] e^{(ik-k_1)l}\right\}\tau \quad \ldots(150)$$

$$a_I \overset{)}{=} \left\{\left[\frac{1}{2} - \frac{i}{4}\left(\frac{k}{k_1} - \frac{k_1}{k}\right)\right] e^{(ik+k_1)l} + \left[\frac{1}{2} - \frac{i}{4}\left(\frac{k_1}{k} - \frac{k}{k_1}\right)\right] e^{(ik-k_1)l}\right\}\tau \quad \ldots(151)$$

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$$\left\{\left[\frac{1}{2} + \frac{i}{4}\left(\frac{k_1}{k} - \frac{k}{k_1}\right)\right] e^{(ik+k_1)l} + \left[\frac{1}{2} - \frac{i}{4}\left(\frac{k_1}{k} - \frac{k}{k_1}\right)\right] e^{(ik-k_1)l}\right\}\tau \quad \ldots(152)$$

$$\frac{a_I}{a_{III}} = \left\{\left[\frac{1}{2} + \frac{i}{4}\left(\frac{k_1}{k} - \frac{k}{k_1}\right)\right] e^{(ik+k_1)l} + \left[\frac{1}{2} - \frac{i}{4}\left(\frac{k_1}{k} - \frac{k}{k_1}\right)\right] e^{(ik-k_1)l} \right\}\tau \div \tau \ldots (153)$$

$$\frac{a_I}{a_{III}} = \left[\frac{1}{2} + \frac{i}{4}\left(\frac{k_1}{k} - \frac{k}{k_1}\right)\right] e^{(ik+k_1)l} + \left[\frac{1}{2} - \frac{i}{4}\left(\frac{k_1}{k} - \frac{k}{k_1}\right)\right] e^{(ik-k_1)l} \ldots (154)$$

This ratio is complex its complex conjugate is

$$\left(\frac{a_I}{a_{III}}\right)^* = \left[\frac{1}{2} - \frac{i}{4}\left(\frac{k_1}{k} - \frac{k}{k_1}\right)\right] e^{(-ik+k_1)l} + \left[\frac{1}{2} + \frac{i}{4}\left(\frac{k_1}{k} - \frac{k}{k_1}\right)\right] e^{(-ik-k_1)l} \ldots (155)$$

$$\left(\frac{a_I}{a_{III}}\right)\left(\frac{a_I}{a_{III}}\right)^* = \left[\frac{1}{2} + \frac{i}{4}\left(\frac{k_1}{k} - \frac{k}{k_1}\right)\right] e^{(ik+k_1)l} + \left[\frac{1}{2} - \frac{i}{4}\left(\frac{k_1}{k} - \frac{k}{k_1}\right)\right] e^{(ik-k_1)l} * \left[\frac{1}{2} - \frac{i}{4}\left(\frac{k_1}{k} - \frac{k}{k_1}\right)\right] e^{(-ik+k_1)l} + \left[\frac{1}{2} + \frac{i}{4}\left(\frac{k_1}{k} - \frac{k}{k_1}\right)\right] e^{(-ik-k_1)l} \ldots (156)$$

$$\left(\frac{a_I}{a_{III}}\right)\left(\frac{a_I}{a_{III}}\right)^* = \left[\left\{\frac{1}{2} + \frac{i}{4}\left(\frac{k_1}{k} - \frac{k}{k_1}\right)\right\}\left\{\frac{1}{2} - \frac{i}{4}\left(\frac{k_1}{k} - \frac{k}{k_1}\right)\right\}\right] e^{2k_1 l} + \left[\left\{\frac{1}{2} - \frac{i}{4}\left(\frac{k_1}{k} - \frac{k}{k_1}\right)\right\}\left\{\frac{1}{2} + \frac{i}{4}\left(\frac{k_1}{k} - \frac{k}{k_1}\right)\right\}\right] e^{-2k_1 l} + \left[\left\{\frac{1}{2} + \frac{i}{4}\left(\frac{k_1}{k} - \frac{k}{k_1}\right)\right\}^2 \left\{\frac{1}{2} - \frac{i}{4}\left(\frac{k_1}{k} - \frac{k}{k_1}\right)\right\}^2\right] e^{0l} \ldots (157)$$

$$\left(\frac{a_I}{a_{III}}\right)\left(\frac{a_I}{a_{III}}\right)^* = \left[\left(\frac{1}{2}\right)^2 - \left\{\frac{i}{4}\left(\frac{k_1}{k} - \frac{k}{k_1}\right)\right\}^2\right] e^{2k_1 l} + \left[\left(\frac{1}{2}\right)^2 - \left\{\frac{i}{4}\left(\frac{k_1}{k} - \frac{k}{k_1}\right)\right\}^2\right] e^{-2k_1 l} + \left[\left\{\frac{1}{2} + \frac{i}{4}\left(\frac{k_1}{k} - \frac{k}{k_1}\right)\right\}^2 \left\{\frac{1}{2} - \frac{i}{4}\left(\frac{k_1}{k} - \frac{k}{k_1}\right)\right\}^2\right] e^{0l} \dots (158)$$

$$\left(\frac{a_I}{a_{III}}\right)\left(\frac{a_I}{a_{III}}\right)^* = \left[\frac{1}{4} - \left\{\frac{-1}{16}\left[\left(\frac{k_1}{k}\right)^2 + \left(\frac{k}{k_1}\right)^2 - 2\right]\right\}\right] e^{2k_1 l} + \left[\frac{1}{4} - \left\{\frac{-1}{16}\left[\left(\frac{k_1}{k}\right)^2 + \left(\frac{k}{k_1}\right)^2 - 2\right]\right\}\right] e^{-2k_1 l} + \left\{\frac{1}{4} + \left[\frac{-1}{16}\left[\left(\frac{k_1}{k}\right)^2 + \left(\frac{k}{k_1}\right)^2 - 2\right] + \frac{i}{4}\left[\frac{k_1}{k} - \frac{k}{k_1}\right]\right]\right\} + \left\{\frac{1}{4} - \left[\frac{-1}{16}\left[\left(\frac{k_1}{k}\right)^2 + \left(\frac{k}{k_1}\right)^2 - 2\right] - \frac{i}{4}\left[\frac{k_1}{k} - \frac{k}{k_1}\right]\right]\right\} \dots (152)$$

$$\left(\frac{a_I}{a_{III}}\right)\left(\frac{a_I}{a_{III}}\right)^* = \left[\frac{1}{4} + \frac{1}{16}\left(\frac{k_1}{k}\right)^2 + \frac{1}{16}\left(\frac{k}{k_1}\right)^2 - \frac{1}{8}\right] e^{2k_1 l} + \left[\frac{1}{4} + \frac{1}{16}\left(\frac{k_1}{k}\right)^2 + \frac{1}{16}\left(\frac{k}{k_1}\right)^2 - \frac{1}{8}\right] e^{-2k_1 l} + \frac{1}{2} \dots (159)$$

$$\left(\frac{a_I}{a_{III}}\right)\left(\frac{a_I}{a_{III}}\right)^* = \left[\frac{1}{8} + \frac{1}{16}\left(\frac{k_1}{k}\right)^2 + \frac{1}{16}\left(\frac{k}{k_1}\right)^2\right] e^{2k_1 l} + \left[\frac{1}{8} + \frac{1}{16}\left(\frac{k_1}{k}\right)^2 + \frac{1}{16}\left(\frac{k}{k_1}\right)^2\right] e^{-2k_1 l} + \frac{1}{2} \dots (160)$$

$$\left(\frac{a_I}{a_{III}}\right)\left(\frac{a_I}{a_{III}}\right)^* = \frac{1}{16}\left[2 + \frac{1}{16}\left(\frac{k_1}{k}\right)^2 + \frac{1}{16}\left(\frac{k}{k_1}\right)^2\right] e^{2k_1 l} + \frac{1}{16}\left[2 + \frac{1}{16}\left(\frac{k_1}{k}\right)^2 + \frac{1}{16}\left(\frac{k}{k_1}\right)^2\right] e^{-2k_1 l} + \frac{1}{2} \dots (161)$$

$$\left(\frac{a_I}{a_{III}}\right)\left(\frac{a_I}{a_{III}}\right)^* = \frac{1}{16}\left[2 + \left(\frac{k_1}{k}\right)^2 + \left(\frac{k}{k_1}\right)^2\right] [e^{2k_1 l} + e^{-2k_1 l}] + \frac{1}{2} \dots (162)$$

$$\left(\frac{a_I}{a_{III}}\right)\left(\frac{a_I}{a_{III}}\right)^* = \frac{1}{16}\left[2 + \left(\frac{k_1}{k}\right)^2 + \left(\frac{k}{k_1}\right)^2\right] [e^{2k_1 l} + e^{-2k_1 l}] + \frac{1}{2} + \frac{1}{2} - \frac{1}{2} \dots (163)$$

$$\left(\frac{a_I}{a_{III}}\right)\left(\frac{a_I}{a_{III}}\right)^* = \frac{1}{16}\left[2 + \left(\frac{k_1}{k}\right)^2 + \left(\frac{k}{k_1}\right)^2\right][e^{2k_1 l} + e^{-2k_1 l}] - \frac{1}{2} + \frac{1}{2} + \frac{1}{2} \quad \ldots (164)$$

$$\left(\frac{a_I}{a_{III}}\right)\left(\frac{a_I}{a_{III}}\right)^* = \frac{1}{16}\left[2 + \left(\frac{k_1}{k}\right)^2 + \left(\frac{k}{k_1}\right)^2\right][e^{2k_1 l} + e^{-2k_1 l}] - \frac{1}{2} + 1 \quad \ldots (165)$$

$$\left(\frac{a_I}{a_{III}}\right)\left(\frac{a_I}{a_{III}}\right)^* = \frac{1}{4}\left[2 + \left(\frac{k_1}{k}\right)^2 + \left(\frac{k}{k_1}\right)^2\right]\frac{1}{4}[e^{2k_1 l} + e^{-2k_1 l}] - \frac{1}{2} + 1 \quad \ldots (166)$$

$$\frac{1}{4}[e^{2k_1 l} + e^{-2k_1 l}] - \frac{1}{2} = sinh^2(k_1 l) \quad \ldots (167)$$

$$\left(\frac{a_I}{a_{III}}\right)\left(\frac{a_I}{a_{III}}\right)^* = \frac{1}{4}\left[2 + \left(\frac{k_1}{k}\right)^2 + \left(\frac{k}{k_1}\right)^2\right] sinh^2(k_1 l) + 1 \quad \ldots (168)$$

$$\left(\frac{k_1}{k}\right)^2 = \frac{2\mu}{\hbar^2}[V(r) - E_T] \div \frac{2\mu}{\hbar^2}[E_T] \quad \ldots (169)$$

$$\left(\frac{k_1}{k}\right)^2 = \frac{V(r) - E_T}{E_T} \quad \ldots (170)$$

$$\left(\frac{k}{k_1}\right)^2 = \frac{E_T}{V(r) - E_T} \quad \ldots (171)$$

$$\left(\frac{a_I}{a_{III}}\right)\left(\frac{a_I}{a_{III}}\right)^* = \frac{1}{4}\left[2 + \frac{V(r) - E_T}{E_T}\right.$$

$$+\frac{E_T}{V(r)-E_T}\Big]\sinh^2(k_1 l)+1 \ldots (172)$$

$$\left(\frac{a_I}{a_{III}}\right)\left(\frac{a_I}{a_{III}}\right)^* = \frac{1}{4}\left[\frac{2E_T(V(r)-E_T)+(V(r)-E_T)^2+{E_T}^2}{E_T(V(r)-E_T)}\right]\sinh^2(k_1 l)+1 \ldots (173)$$

$$\left(\frac{a_I}{a_{III}}\right)\left(\frac{a_I}{a_{III}}\right)^* = \frac{1}{4}\left[\frac{{E_T}^2}{E_T(V(r)-E_T)}\right]\sinh^2(k_1 l)+1 \ldots (174)$$

But

$$\sinh(k_1 l)=\frac{1}{2}[e^{k_1 l}-e^{-k_1 l}] \ldots (175)$$

Which for $k_1 l \gg 1$

$i.e.\ k_1 l \to \infty$

$$e^{-k_1 l}=0$$

$$\sinh(k_1 l)=\frac{1}{2}e^{k_1 l} \ldots (176)$$

$$\sinh^2(k_1 l)=\frac{1}{4}e^{2k_1 l} \ldots (177)$$

$$\left(\frac{a_I}{a_{III}}\right)\left(\frac{a_I}{a_{III}}\right)^* = \frac{1}{4}\left[\frac{{E_T}^2}{E_T(V(r)-E_T)}\right]\frac{1}{4}e^{2k_1 l}+1 \ldots (178)$$

$$\left(\frac{a_I}{a_{III}}\right)\left(\frac{a_I}{a_{III}}\right)^* = \frac{1}{16}\left[\frac{{E_T}^2}{E_T(V(r)-E_T)}\right]e^{2k_1 l}+1 \ldots (179)$$

$$\left(\frac{a_I}{a_{III}}\right)\left(\frac{a_I}{a_{III}}\right)^* = \frac{1}{16}\left[\frac{{E_T}^2}{E_T(V(r)-E_T)}\right]e^{2k_1 l}+\frac{e^{2k_1 l}}{e^{2k_1 l}} \ldots (180)$$

$$\left(\frac{a_I}{a_{III}}\right)\left(\frac{a_I}{a_{III}}\right)^* = \left[\frac{{E_T}^2}{16E_T(V(r)-E_T)}+\frac{1}{e^{2k_1 l}}\right]e^{2k_1 l} \ldots (181)$$

$$\left(\frac{a_I}{a_{III}}\right)\left(\frac{a_I}{a_{III}}\right)^* = \left[\frac{{E_T}^2}{16E_T(V(r)-E_T)}+e^{-2k_1 l}\right]e^{2k_1 l} \ldots (182)$$

$$\left(\frac{a_I}{a_{III}}\right)\left(\frac{a_I}{a_{III}}\right)^* = \left[\frac{{E_T}^2}{16E_T(V(r) - E_T)} + 0\right] e^{2k_1 l} \ldots (183)$$

$$\left(\frac{a_I}{a_{III}}\right)\left(\frac{a_I}{a_{III}}\right)^* = \left[\frac{{E_T}^2}{16\left(E_T V(r) - {E_T}^2\right)}\right] e^{2k_1 l} \ldots (184)$$

$$\left(\frac{a_I}{a_{III}}\right)\left(\frac{a_I}{a_{III}}\right)^* = \frac{{E_T}^2}{16\left(E_T V(r) - {E_T}^2\right)} e^{2k_1 l} \ldots (185)$$

$$\left|\frac{a_I}{a_{III}}\right|^2 = \frac{{E_T}^2}{16\left(E_T V(r) - {E_T}^2\right)} e^{2k_1 l} \ldots (186)$$

$$\left|\frac{a_{III}}{a_I}\right|^2 = \frac{16\left(E_T V(r) - {E_T}^2\right)}{{E_T}^2} e^{-2k_1 l} \ldots (187)$$

From equation (98)

$$T_\alpha = \frac{16\left(E_T V(r) - {E_T}^2\right)}{{E_T}^2} e^{-2k_1 l} \ldots (188)$$

Important factor in most physical cases is exponential, usually the term in front of the exponential ignored

$$T_\alpha = e^{-2k_1 l} \ldots (189)$$

Let

$G = k_1 l$, G is Gamow factor

$$T_\alpha = e^{-2G} \ldots (190)$$

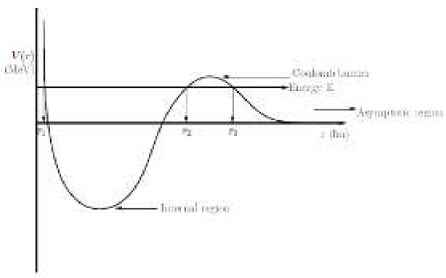
2.5. Determination Gamow factor: From equation (58)

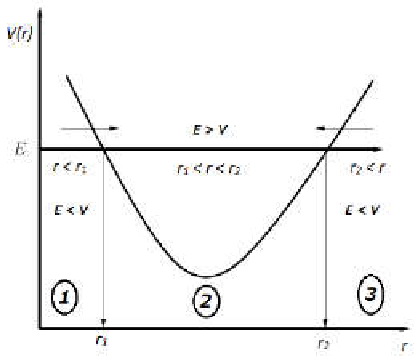
$$G = \sqrt{\frac{2\mu}{\hbar^2}[V(r) - E_T]}\, l \quad \ldots (191)$$

If $V\,(r)$ is not constant, i.e. $V(r)$ varies with r

$$G = \int_{r_1}^{r_2} \sqrt{\frac{2\mu}{\hbar^2}[V(r) - E_T]}\, dr \quad \ldots (192)$$

Where r_1, r_2 are the classical turning points, i.e.points at which $ET=V\,(r)$





If barrier is broken up between r_1, r_2 into $n$adjacent barriers of thickness Δr

$$r_2 - r_1 = n\Delta r \quad \dots(193)$$

Total transmission coefficient

$$T_\alpha = T_1 T_2 \dots T_n \quad \dots(194)$$

$$T_\alpha = e^{-2G_1} e^{-2G_2} \dots e^{-2G_n} \quad \dots(195)$$

$$T_\alpha = e^{-2(G_1 G_2 \dots G_n)} \quad \dots(196)$$

As $n \to \infty$

$$T_\alpha = e^{-2G} \dots(197)$$

$$V(r) = V_C(r) = \frac{Z_c Z_d e^2}{r} \dots(198)$$

Where Z_c, Z_d are atomic numbers of cluster and core nuclei, respectively.

$$r_1 = r_{eff}$$

$$r_2 = l$$

$$E_T = Q_\alpha \quad \dots(199)$$

From equation (192) and (198)

$$2G = 2\int_{r_{eff}}^{l} \sqrt{\frac{2\mu}{\hbar^2}\left[\frac{Z_c Z_d e^2}{r} - Q_\alpha\right]}\, dr \quad \dots(200)$$

$$2G = \frac{2}{\hbar}\int_{r_{eff}}^{l} \sqrt{2\mu Q_\alpha \left[\frac{Z_c Z_d e^2}{Q_\alpha}\frac{1}{r} - 1\right]}\, dr \quad \dots(201)$$

At $r = l$

$$Q_\alpha = V(r = l) \dots(202)$$

$$Q_\alpha = \frac{Z_c Z_d e^2}{l} \dots(203)$$

$$l = \frac{Z_c Z_d e^2}{Q_\alpha} \dots(204)$$

$$2G = \frac{2}{\hbar}\int_{r_{eff}}^{l} \sqrt{2\mu Q_\alpha \left[l\frac{1}{r} - 1\right]}\, dr \quad \dots(205)$$

$$2G = \frac{2\sqrt{2\mu Q_\alpha}}{\hbar}\int_{r_{eff}}^{l} \left(\frac{l}{r} - 1\right)^{\frac{1}{2}} dr \quad \dots(206)$$

$$2G = \frac{2\sqrt{2\mu Q_\alpha}}{\hbar}\int_{r_{eff}}^{l} \left(\frac{l}{r}\left[1 - \frac{r}{l}\right]\right)^{\frac{1}{2}} dr \quad \dots(206)$$

$$2G = \frac{2\sqrt{2\mu Q_\alpha}}{\hbar}\int_{r_{eff}}^{l} \left(\frac{l}{r}\right)^{\frac{1}{2}} \left(1 - \frac{r}{l}\right)^{\frac{1}{2}} dr \quad \dots(206)$$

$$2G = \frac{2\sqrt{2\mu Q_\alpha}}{\hbar}\int_{r_{eff}}^{l} \left(\frac{r}{l}\right)^{-\frac{1}{2}} \left(1 - \frac{r}{l}\right)^{\frac{1}{2}} dr \quad \dots(206)$$

$2G$

$$= \frac{2\sqrt{2\mu Q_\alpha}}{\hbar} \int_{r_{eff}}^{l} \frac{\left(1-\frac{r}{l}\right)^{\frac{1}{2}}}{\left(\frac{r}{l}\right)^{\frac{1}{2}}} dr \quad \dots (206)$$

$$2G = \frac{2\sqrt{2\mu Q_\alpha}}{\hbar} \int_{r_{eff}}^{l} \frac{\sqrt{1-\frac{r}{l}}}{\sqrt{\frac{r}{l}}} dr \quad \dots (206)$$

The integral involve of the form $\sqrt{a^2 - x^2}$ he solution by trigonometric substitutions as follows

Let

$$a^2 = 1$$
$$a = 1$$

And

$$x^2 = \frac{r}{l}$$
$$2xdx = \frac{dr}{l}$$
$$2lxdx = dr$$
$$x = \sqrt{\frac{r}{l}}$$

Let

$$x = asin\theta$$
$$x = sin\theta$$
$$dx = cos\theta\, d\theta$$
$$\theta = sin^{-1}x$$
$$dr = 2lsin\theta cos\theta\, d\theta$$
$$\sqrt{1-\frac{r}{l}} = \sqrt{1-x^2} = \sqrt{1-sin^2\theta}$$
$$= \sqrt{cos^2\theta} = cos\theta$$

$$2G = \frac{2\sqrt{2\mu Q_\alpha}}{\hbar} \left[\int_{r_{eff}}^{l} \frac{cos\theta(2lsin\theta cos\theta\, d\theta)}{sin\theta} \right] \quad \dots (207)$$

$$2G = \frac{2\sqrt{2\mu Q_\alpha}}{\hbar} \left[2l \int_{r_{eff}}^{l} cos^2\theta d\theta \right] \quad \dots (208)$$

$$2G = \frac{2\sqrt{2\mu Q_\alpha}}{\hbar} \left[2l \int_{r_{eff}}^{l} \left(\frac{1+cos2\theta}{2}\right) d\theta \right] \quad \dots (209)$$

$$2G = \frac{2\sqrt{2\mu Q_\alpha}}{\hbar} \left[l \int_{r_{eff}}^{l} (1 + cos2\theta) d\theta \right] \quad \dots (210)$$

$$2G = \frac{2\sqrt{2\mu Q_\alpha}}{\hbar}\left[l\left(\theta + \frac{1}{2}sin2\theta\Big|_{r_{eff}}^{l}\right)\right] \quad ...(211)$$

$$2G = \frac{2\sqrt{2\mu Q_\alpha}}{\hbar}\left[l\left(\theta + \frac{1}{2}\{2sin\theta cos\theta\}\Big|_{r_{eff}}^{l}\right)\right] \quad ...(212)$$

$$2G = \frac{2\sqrt{2\mu Q_\alpha}}{\hbar}\left[l\left(\theta + sin\theta cos\theta|_{r_{eff}}^{l}\right)\right] \quad ...(213)$$

$$2G = \frac{2\sqrt{2\mu Q_\alpha}}{\hbar}\left[l\left(sin^{-1}x + x\sqrt{1-x^2}\Big|_{r_{eff}}^{l}\right)\right] \quad ...(214)$$

$$2G = \frac{2\sqrt{2\mu Q_\alpha}}{\hbar}\left[l\left(sin^{-1}\left(\sqrt{\frac{r}{l}}\right) + \sqrt{\frac{r}{l}}\sqrt{1-\frac{r}{l}}\Big|_{r_{eff}}^{l}\right)\right] \quad ...(215)$$

$$2G = \frac{2\sqrt{2\mu Q_\alpha}}{\hbar}\left\{l\left[sin^{-1}\left(\sqrt{\frac{l}{l}}\right) + \sqrt{\frac{l}{l}}\sqrt{1-\frac{l}{l}} - \left[sin^{-1}\left(\sqrt{\frac{r_{eff}}{l}}\right) + \sqrt{\frac{r_{eff}}{l}}\sqrt{1-\frac{r_{eff}}{l}}\right]\right]\right\} \quad ...(216)$$

$$2G = \frac{2\sqrt{2\mu Q_\alpha}}{\hbar}\left\{l\left[sin^{-1}(1) + 1\sqrt{1-1} - \left[sin^{-1}\left(\sqrt{\frac{r_{eff}}{l}}\right) + \sqrt{\frac{r_{eff}}{l}}\sqrt{1-\frac{r_{eff}}{l}}\right]\right]\right\} \quad ...(217)$$

$$2G = \frac{2\sqrt{2\mu Q_\alpha}}{\hbar}\left\{l\left[sin^{-1}(1) + 0 - \left[sin^{-1}\left(\sqrt{\frac{r_{eff}}{l}}\right) + \sqrt{\frac{r_{eff}}{l}}\sqrt{1-\frac{r_{eff}}{l}}\right]\right]\right\} \quad ...(218)$$

$$2G = \frac{2\sqrt{2\mu Q_\alpha}}{\hbar}\left\{l\left[\frac{\pi}{2} - sin^{-1}\left(\sqrt{\frac{r_{eff}}{l}}\right) - \sqrt{\frac{r_{eff}}{l}}\sqrt{1-\frac{r_{eff}}{l}}\right]\right\} \quad ...(219)$$

$$2G = \frac{2\sqrt{2\mu Q_\alpha}}{\hbar}\left\{l\left[\left\{\frac{\pi}{2} - sin^{-1}\left(\sqrt{\frac{r_{eff}}{l}}\right)\right\} - \sqrt{\frac{r_{eff}}{l}}\sqrt{1-\frac{r_{eff}}{l}}\right]\right\} \quad ...(220)$$

$$2G = \frac{2\sqrt{2\mu Q_\alpha}}{\hbar}\left[l\left(cos^{-1}\left(\sqrt{\frac{r_{eff}}{l}}\right) - \sqrt{\frac{r_{eff}}{l}}\sqrt{1-\frac{r_{eff}}{l}}\right)\right] \quad ...(221)$$

But

$$\frac{r_{eff}}{l} \ll 1$$

$$\sqrt{\frac{r_{eff}}{l}} \ll 1$$

As $\left|\sqrt{\frac{r_{eff}}{l}}\right| \ll 1$

$$cos^{-1}\left(\sqrt{\frac{r_{eff}}{l}}\right) = \frac{\pi}{2} - \sum_{j=0}^{\infty}\frac{1}{4^j}\frac{(2j)!!}{j!!}\frac{1}{(2j+1)}\left(\sqrt{\frac{r_{eff}}{l}}\right)^{2j+1} \quad ...(222)$$

$$cos^{-1}\left(\sqrt{\frac{r_{eff}}{l}}\right) = \frac{\pi}{2} - \left[\left(\sqrt{\frac{r_{eff}}{l}}\right)^1 + \cdots\right] \quad ...(223)$$

$$cos^{-1}\left(\sqrt{\frac{r_{eff}}{l}}\right) = \frac{\pi}{2} - \sqrt{\frac{r_{eff}}{l}} \quad ...(224)$$

$$\left(1+\frac{r_{eff}}{l}\right)^{\frac{1}{2}} = \sum_{j=0}^{\infty}\frac{(-1)^j}{4^j}\frac{(2j)!!}{j!!}\frac{1}{(1-2j)}\left(\frac{r_{eff}}{l}\right)^j \quad ...(225)$$

$$\left(1+\frac{r_{eff}}{l}\right)^{\frac{1}{2}} = 1 + \frac{1}{2}\frac{r_{eff}}{l} + \cdots \quad ...(226)$$

$$\left(1-\frac{r_{eff}}{l}\right)^{\frac{1}{2}} = 1 - \frac{1}{2}\frac{r_{eff}}{l} + \cdots \quad ...(227)$$

$$\sqrt{\frac{r_{eff}}{l}}\sqrt{1-\frac{r_{eff}}{l}} = \left(\frac{r_{eff}}{l}\right)^{\frac{1}{2}}\left[1 - \frac{1}{2}\frac{r_{eff}}{l} + \cdots\right] \quad ...(228)$$

$$\sqrt{\frac{r_{eff}}{l}}\sqrt{1-\frac{r_{eff}}{l}} = \left(\frac{r_{eff}}{l}\right)^{\frac{1}{2}} - \frac{1}{2}\left(\frac{r_{eff}}{l}\right)^{\frac{3}{2}} + \cdots \quad \dots(229)$$

$$\sqrt{\frac{r_{eff}}{l}}\sqrt{1-\frac{r_{eff}}{l}} = \left(\frac{r_{eff}}{l}\right)^{\frac{1}{2}} \dots(230)$$

$$2G = \frac{2\sqrt{2\mu Q_\alpha}}{\hbar}\left[l\left(\frac{\pi}{2} - \sqrt{\frac{r_{eff}}{l}} - \left(\frac{r_{eff}}{l}\right)^{\frac{1}{2}}\right)\right] \quad \dots(231)$$

$$2G = \frac{2\sqrt{2\mu Q_\alpha}}{\hbar}\left[l\left(\frac{\pi}{2} - 2\sqrt{\frac{r_{eff}}{l}}\right)\right] \quad \dots(232)$$

$$2G = \frac{2l}{\hbar}\sqrt{2\mu Q_\alpha}\left[\left(\frac{\pi}{2} - 2\sqrt{\frac{r_{eff}}{l}}\right)\right] \quad \dots(233)$$

From equation (204)

$$2G = \frac{2}{\hbar}\frac{Z_c Z_d e^2}{Q_\alpha}\sqrt{2\mu Q_\alpha}\left[\left(\frac{\pi}{2} - 2\sqrt{\frac{Q_\alpha r_{eff}}{Z_c Z_d e^2}}\right)\right] \quad \dots(234)$$

$$2G = 2Z_c Z_d e^2\sqrt{\frac{2\mu Q_\alpha}{Q_\alpha^2 \hbar^2}}\left[\left(\frac{\pi}{2} - 2\sqrt{\frac{Q_\alpha r_{eff}}{Z_c Z_d e^2}}\right)\right] \quad \dots(235)$$

$$2G = 2Z_c Z_d e^2\sqrt{\frac{2\mu}{Q_\alpha \hbar^2}}\left[\left(\frac{\pi}{2} - 2\sqrt{\frac{Q_\alpha}{V(r)}}\right)\right] \quad \dots(236)$$

$$2G = 2Z_c Z_d e^2\sqrt{\frac{2\mu}{\hbar^2}}\sqrt{\frac{1}{Q_\alpha}}\left[\left(\frac{\pi}{2} - 2\sqrt{\frac{Q_\alpha}{V(r)}}\right)\right] \quad \dots(237)$$

Where $\frac{Q_\alpha}{V(r)} \ll 1, \pi = 3.14$

$$T_\alpha = e^{-2Z_c Z_d e^2\sqrt{\frac{2\mu}{\hbar^2}}\sqrt{\frac{1}{Q_\alpha}}\left[\left(\frac{\pi}{2}-2\sqrt{\frac{Q_\alpha}{V(r)}}\right)\right]} \dots(238)$$

From equations (17) and (238) into equation (3)

$$\lambda_\alpha = \frac{\sqrt{\frac{2(Q_\alpha+V(r))C^2}{\mu}}}{2r_{eff}} e^{-2Z_cZ_de^2\sqrt{\frac{2\mu}{\hbar^2}}\sqrt{\frac{1}{Q_\alpha}}\left[\left(\frac{\pi}{2}-2\sqrt{\frac{Q_\alpha}{V(r)}}\right)\right]} \quad ...(239)$$

$$log\lambda_\alpha = log\left[\frac{\sqrt{\frac{2(Q_\alpha+V(r))C^2}{\mu}}}{2r_{eff}} e^{-2Z_cZ_de^2\sqrt{\frac{2\mu}{\hbar^2}}\sqrt{\frac{1}{Q_\alpha}}\left[\left(\frac{\pi}{2}-2\sqrt{\frac{Q_\alpha}{V(r)}}\right)\right]}\right]$$

$$log\lambda_\alpha = log\left[e^{-2Z_cZ_de^2\sqrt{\frac{2\mu}{\hbar^2}}\sqrt{\frac{1}{Q_\alpha}}\left[\left(\frac{\pi}{2}-2\sqrt{\frac{Q_\alpha}{V(r)}}\right)\right]}\right] + log\left[\frac{\sqrt{\frac{2(Q_\alpha+V(r))C^2}{\mu}}}{2r_{eff}}\right] \quad ...(241)$$

$$log\lambda_\alpha = -2Z_cZ_de^2\sqrt{\frac{2\mu}{\hbar^2}}\sqrt{\frac{1}{Q_\alpha}}\left[\left(\frac{\pi}{2} - 2\sqrt{\frac{Q_\alpha}{V(r)}}\right)\right] log[e] + log\left[\frac{\sqrt{\frac{2(Q_\alpha+V(r))C^2}{\mu}}}{2r_{eff}}\right] \quad ...(242)$$

But

$$t_{\frac{1}{2}} = \frac{ln2}{\lambda_\alpha} \quad ...(243)$$

$$logt_{\frac{1}{2}} = log\left(\frac{ln2}{\lambda_\alpha}\right) \quad ...(244)$$

$$logt_{\frac{1}{2}} = log(ln2) - log\lambda_\alpha \quad ...(245)$$

$$logt_{\frac{1}{2}} = log(ln2) + 2Z_cZ_de^2\sqrt{\frac{2\mu}{\hbar^2}}\sqrt{\frac{1}{Q_\alpha}}\left[\left(\frac{\pi}{2} - 2\sqrt{\frac{Q_\alpha}{V(r)}}\right)\right] log[e] - log\left[\frac{\sqrt{\frac{2(Q_\alpha+V(r))C^2}{\mu}}}{2r_{eff}}\right] \quad ...(246)$$

$$\log t_{\frac{1}{2}} = 2Z_cZ_de^2\sqrt{\frac{2\mu}{\hbar^2}}\sqrt{\frac{1}{Q_\alpha}}\left[\left(\frac{\pi}{2} - 2\sqrt{\frac{Q_\alpha}{V(r)}}\right)\right]\log[e] + \log(ln2) - \log\left[\frac{\sqrt{\frac{2(Q_\alpha + V(r))C^2}{\mu}}}{2r_{eff}}\right] \quad \ldots (247)$$

$$\log t_{\frac{1}{2}} = 2Z_cZ_de^2\sqrt{\frac{2\mu}{\hbar^2}}\sqrt{\frac{1}{Q_\alpha}}\left[\left(\frac{\pi}{2} - 2\sqrt{\frac{Q_\alpha}{V(r)}}\right)\right]\log[e] + \log\left[ln2 \div \frac{\sqrt{\frac{2(Q_\alpha + V(r))C^2}{\mu}}}{2r_{eff}}\right] \quad \ldots (248)$$

$$\log t_{\frac{1}{2}} = 2Z_cZ_de^2\sqrt{\frac{2\mu}{\hbar^2}}\sqrt{\frac{1}{Q_\alpha}}\left[\left(\frac{\pi}{2} - 2\sqrt{\frac{Q_\alpha}{V(r)}}\right)\right]\log[e] + \log\left[\frac{2r_{eff}ln2}{\sqrt{\frac{2(Q_\alpha + V(r))C^2}{\mu}}}\right] \quad \ldots (249)$$

$$\log t_{\frac{1}{2}} = \pi\log[e]Z_cZ_de^2\sqrt{\frac{2\mu}{\hbar^2}}\sqrt{\frac{1}{Q_\alpha}} - 4\log[e]Z_cZ_de^2\sqrt{\frac{2\mu}{\hbar^2V(r)}} + \log\left[\frac{2r_{eff}ln2}{\sqrt{\frac{2(Q_\alpha + V(r))C^2}{\mu}}}\right] \quad \ldots (250)$$

$$Q_\alpha \ll V(r)$$

$$\log t_{\frac{1}{2}} = \pi \log[e] Z_c Z_d e^2 \sqrt{\frac{2\mu}{\hbar^2}} \sqrt{\frac{1}{Q_\alpha}} - 4\log[e] Z_c Z_d e^2 \sqrt{\frac{2\mu}{\hbar^2 V(r)}} + \log\left[\frac{2r_{eff} \ln 2}{\sqrt{\frac{2V(r)C^2}{\mu}}}\right] \quad \dots(251)$$

$$\log t_{\frac{1}{2}} = \left(Z_c Z_d e^2 \sqrt{\frac{2\mu}{\hbar^2}} \pi \log[e]\right) \sqrt{\frac{1}{Q_\alpha}} + \log\left[\frac{2r_{eff} \ln 2}{\sqrt{\frac{2V(r)C^2}{\mu}}}\right] - 4Z_c Z_d e^2 \sqrt{\frac{2\mu}{\hbar^2 V(r)}} \log[e] \quad \dots(252)$$

$$\log t_{\frac{1}{2}} = C_3 \sqrt{\frac{1}{Q_\alpha}} + C_4 \quad \dots(253)$$

Where

$$C_3 = Z_c Z_d \pi e^2 \sqrt{\frac{2\mu}{\hbar^2}} \log[e] \quad \dots(254)$$

$$C_4 = \log\left[\frac{2r_{eff} \ln 2}{\sqrt{\frac{2V(r)C^2}{\mu}}}\right] - 4Z_c Z_d e^2 \sqrt{\frac{2\mu}{\hbar^2 V(r)}} \log[e] \quad \dots(255)$$